

Solutions – Day 1

1.) (a.) The length and widths of M1 is $2\mu\text{m}$ and $10\mu\text{m}$ and of M2 is $2\mu\text{m}$ and $20\mu\text{m}$. Therefore,

$$C_{gd1} = 220 \times 10^{-12} \cdot 10 \times 10^{-6}$$

$$C_{gd1} = \underline{2.2\text{fF}}$$

$$C_{gd2} = 220 \times 10^{-12} \cdot 20 \times 10^{-6}$$

$$C_{gd1} = \underline{4.4\text{fF}}$$

Next, we must find the area and perimeter of each drain.

$$AD1 = 60\mu\text{m}^2 \text{ \& } PD1 = 32\mu\text{m}$$

$$AD2 = 120\mu\text{m}^2 \text{ \& } PD1 = 52\mu\text{m}$$

$$C_{bd1} = \frac{CJ \cdot AD1}{\left(1 + \frac{2.5V}{2|\phi_F|}\right)^{MJ}} + \frac{CJSW \cdot PD1}{\left(1 + \frac{2.5V}{2|\phi_F|}\right)^{MJSW}} = \frac{770 \times 10^{-6} \cdot 60 \times 10^{-12}}{\left(1 + \frac{2.5V}{0.8}\right)^{0.5}} + \frac{380 \times 10^{-12} \cdot 32 \times 10^{-6}}{\left(1 + \frac{2.5V}{0.8}\right)^{0.38}}$$

$$C_{bd1} = 22.75\text{fF} + 7.10\text{fF} = \underline{29.84\text{fF}}$$

$$C_{bd2} = \frac{CJ \cdot AD2}{\left(1 + \frac{2.5V}{2|\phi_F|}\right)^{MJ}} + \frac{CJSW \cdot PD2}{\left(1 + \frac{2.5V}{2|\phi_F|}\right)^{MJSW}} = \frac{560 \times 10^{-6} \cdot 120 \times 10^{-12}}{\left(1 + \frac{2.5V}{0.7}\right)^{0.5}} + \frac{350 \times 10^{-12} \cdot 52 \times 10^{-6}}{\left(1 + \frac{2.5V}{0.7}\right)^{0.35}}$$

$$C_{bd2} = 31.43\text{fF} + 10.69\text{fF} = \underline{42.12\text{fF}}$$

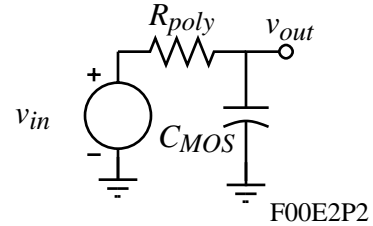
(b.) Assume that the equivalent bulk resistance for the sources is found by the area defined by the rectangle between the edge of the polysilicon and the closest edge of the contacts. This assumption ignores the fact that current at the edges of the rectangle will not be the same as that in the middle. For M1, this rectangle is $2\mu\text{m}$ by $10\mu\text{m}$ and for M2 this rectangle is $2\mu\text{m}$ by $20\mu\text{m}$. Therefore,

$$R_{S1} \approx 35\Omega/\text{sq.} \times (2\mu\text{m}/10\mu\text{m}) + 4\Omega/2 = 7\Omega + 2\Omega = \underline{9\Omega}$$

and

$$R_{S2} \approx 80\Omega/\text{sq.} \times (2\mu\text{m}/20\mu\text{m}) + 4\Omega/4 = 8\Omega + 1\Omega = \underline{9\Omega}$$

2.) A simple first-order filter shown is to be built with a polysilicon resistor and a MOS capacitor. The polysilicon resistor has a sheet resistance of $50\Omega/\text{sq.} \pm 30\%$ and is $5\mu\text{m}$ wide. The MOS capacitor is $2\text{fF}/\mu\text{m}^2 \pm 10\%$. The -3dB frequency of the lowpass filter is 1MHz . (a.) Choose the size of the resistor (the number of squares, N) to minimize the total area of the filter including both the resistor and the capacitor.



Find the area of the resistor and the capacitor in μm^2 and their values. (b.) Using the worst-case tolerance of the resistor and capacitor, find the maximum and minimum -3dB frequencies.

Solution

(a.)

$$\text{Value of } R = 50\Omega/\text{sq.} \times N \text{ sq.} = 50N \Omega$$

$$\text{Value of } C = 2\text{fF}/\mu\text{m}^2 \times A_C \mu\text{m}^2 = 2A_C \text{ fF}$$

$$\text{Area of } C = A_C$$

$$\text{Area of } R = A_R = 25\mu\text{m}^2 \times N = 25N \mu\text{m}^2$$

$$\text{Total Area} = A_T = (25N + A_C) \mu\text{m}^2$$

We know that the RC product is given as

$$RC = \frac{1}{2\pi \times 10^6} = (50N)(2A_C \times 10^{-15}) = NA_C \times 10^{-13}$$

$$\therefore A_C = \frac{1}{2\pi \times 10^{-7} N}$$

$$\text{Thus, } A_T = 25N + \frac{1}{2\pi \times 10^{-7} N} \quad \rightarrow \quad \frac{dA_T}{dN} = 25 - \frac{1}{2\pi \times 10^{-7} N^2} = 0$$

$$\therefore N = \frac{1}{\sqrt{50\pi \times 10^{-7}}} = 252 \quad \Rightarrow \quad \underline{\underline{A_R = 252 \times 25 \mu\text{m}^2 = 6308 \mu\text{m}^2}} \quad \text{and} \quad \underline{\underline{A_C = 6308 \mu\text{m}^2}}$$

$$\text{Also, } \underline{\underline{R_{poly} = R = 252 \times 50 \Omega = 12.6 \text{ k}\Omega}} \quad \text{and} \quad \underline{\underline{C_{MOS} = 6308 \mu\text{m}^2 \times 2 \text{ fF}/\mu\text{m}^2 = 12.6 \text{ pF}}}$$

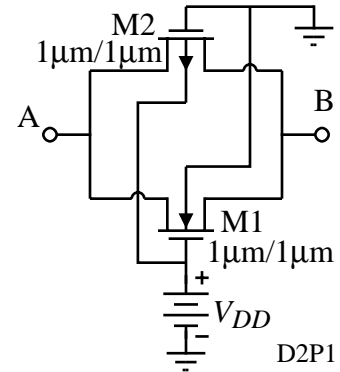
(b.)

$$\text{Maximum } -3\text{dB frequency} = \frac{1}{2\pi(0.7)(12.6 \text{ k}\Omega)(0.9)(12.6 \text{ pF})} = \underline{\underline{1.6 \text{ MHz}}}$$

$$\text{Minimum } -3\text{dB frequency} = \frac{1}{2\pi(1.3)(12.6 \text{ k}\Omega)(1.1)(12.6 \text{ pF})} = \underline{\underline{0.7 \text{ MHz}}}$$

Solutions – Day 2

1.) A transmission gate is shown. If $V_{DD} = 2V$, what is the highest possible large signal ON resistance of the transmission gate. Ignore the bulk effects and assume that $v_{DS} \approx 0V$.



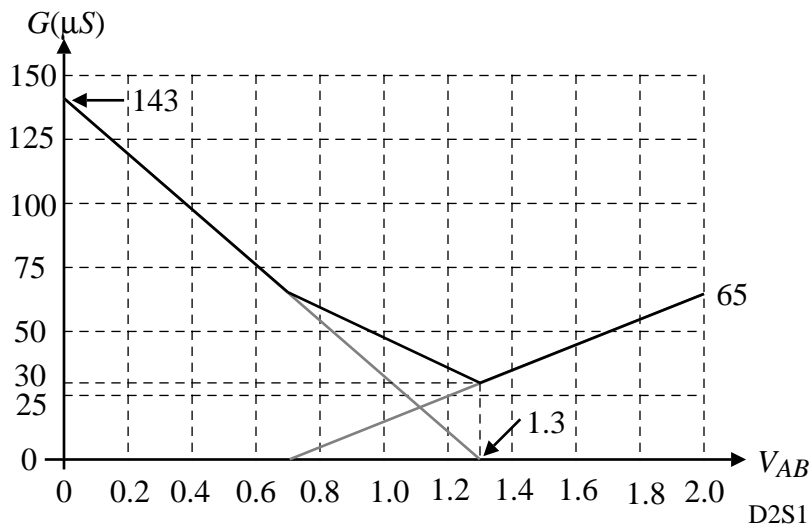
Solution

$$R_{NMOS} = \frac{1}{K_N(W_1/L_1)(V_{GS1}-V_{TN})} = \frac{10^6}{110(2-V_{AB}-0.7)}$$

$$= \frac{10^6}{110(1.3-V_{AB})} \Rightarrow G_{NMOS} = 110(1.3-V_{AB})\mu S$$

$$R_{PMOS} = \frac{1}{K_P(W_2/L_2)(V_{SG2}-|V_{TP}|)} = \frac{10^6}{50(V_{AB}-0.7)} \Rightarrow G_{PMOS} = 50(V_{AB}-0.7)\mu S$$

First, plot the sum of the conductances to graphically locate the minimum value of switch conductance.



We note that the minimum value of switch conductance occurs at $V_{AB} = 1.3V$. This corresponds to $30\mu S$. Inverting this value gives the maximum switch resistance which is $33.3k\Omega$.

Therefore, the maximum ON resistance occurs at $1.3V$ and is $33.3k\Omega$.

$R_{ON(max)} = 33.3k\Omega \text{ at } V_{AB} = 1.3V$

- 2.) (a.) If all W/Ls are $100\mu\text{m}/1\mu\text{m}$, find the value of V_{GG} that will give a minimum value of V_{MIN} .
 (b.) Find the small signal values of R_{in} , R_{out} , and i_{out}/i_{in} .

Solution:

(a.) $V_{GG} = V_{GS4} + V_{DS1}(\text{sat}) = V_{TN} + 2 \cdot V_{DS}(\text{sat})$

$$= 0.70 + 2 \sqrt{\frac{2 \cdot 100\mu\text{A}}{110\mu\text{A}/\text{V}^2 \cdot 100}} = 0.835$$

$V_{MIN} = 2(0.135) = 0.27\text{V}$

(b.) $R_{out} \approx g_{m4} r_{ds4} r_{ds2}$ and $i_{out}/i_{in} \approx 1$

Small signal model for R_{in} calculation:

$$v_{in} = (i_{in} - g_{m3} v_{gs3}) r_{ds3} + (i_{in} - g_{m1} v_{gs1}) r_{ds1}$$

$$v_{gs1} = v_{in} \text{ and } v_{gs3} = -v_{s3} = -(i_{in} - g_{m1} v_{gs1}) r_{ds1}$$

$$= -(i_{in} - g_{m1} v_{in}) r_{ds1}$$

$$\therefore v_{in} = [i_{in} r_{ds3} + g_{m3} r_{ds1} r_{ds3} - g_{m1} g_{m3} r_{ds1} r_{ds3}] + [i_{in} r_{ds1} - g_{m1} r_{ds1} v_{in}]$$

$$\text{or } v_{in} [1 + g_{m1} g_{m3} r_{ds1} r_{ds3} + g_{m1} r_{ds1}] = i_{in} [r_{ds1} + r_{ds3} + g_{m3} r_{ds3}]$$

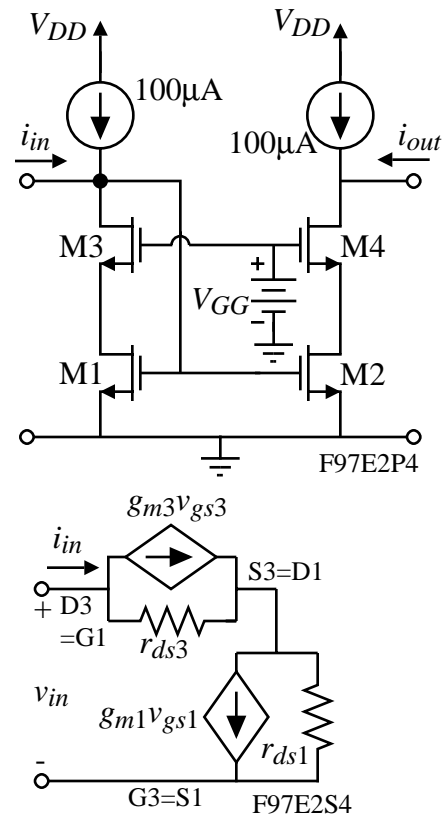
$$\therefore R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_{ds1} + r_{ds3} + g_{m3} r_{ds3}}{1 + g_{m1} g_{m3} r_{ds1} r_{ds3} + g_{m1} r_{ds1}} \approx \frac{1}{g_{m1}}$$

$$g_{m1} = \sqrt{2 \cdot 110 \cdot 100 \cdot 100} = 1.48\text{mS} = g_{m4}, \quad r_{ds1} = r_{ds2} = r_{ds3} = r_{ds4} = 0.25\text{M}\Omega.$$

$$\therefore R_{in} \approx \frac{1}{g_{m1}} = \frac{1}{1.48\text{mS}} = 674\Omega$$

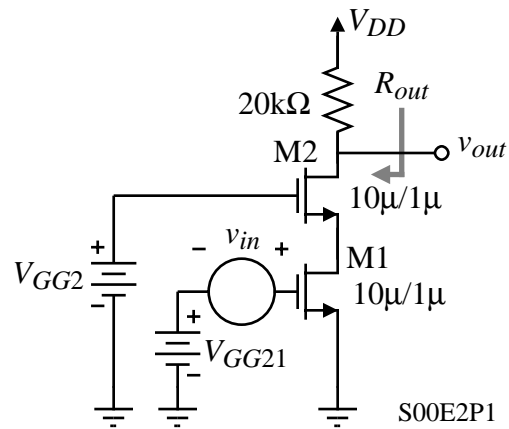
$$R_{out} \approx 250\text{k}\Omega (1.48\text{mS}) 250\text{k}\Omega = 92.5\text{M}\Omega$$

and $\frac{i_{out}}{i_{in}} = 1$



Day 3 - Solutions**Problem 1**

A MOS cascode amplifier is shown below. Assume all transistors are saturated and the $K_N' = 110\mu\text{A}/\text{V}^2$, $V_T = 0.7\text{V}$, and $\lambda_N = 0.04\text{V}^{-1}$. The value of V_{GG1} gives a dc bias current in both transistors of $100\mu\text{A}$. (a.) Find the value of V_{GG2} which gives a minimum output voltage (same as $V_{MIN}(\text{out})$ for current sinks). What is the value of minimum output voltage? (b.) Find the small-signal output resistance, R_{out} . (c.) Find the small-signal voltage gain, v_{out}/v_{in} .

**Solution**

$$(a.) V_{ON} = \sqrt{\frac{2I_D}{K'(W/L)}} = \sqrt{\frac{2 \cdot 100\mu\text{A}}{110\mu\text{A}/\text{V}^2 \cdot 10}} = 0.4264\text{V}$$

$$\therefore V_{GS} = V_{ON} + V_T = 0.4264 + 0.7 = 1.1264\text{V} \quad \Rightarrow \quad \boxed{V_{GG2} = 2V_{ON} + V_T = 1.553\text{V}}$$

$$\text{Also } \boxed{V_{MIN} = 2V_{ON} = 2 \cdot 0.4264 = 0.8528\text{V}}$$

$$(b.) R_{out} = 20\text{k}\Omega \parallel g_m^{-2} r_{ds} \quad g_m = \sqrt{2I_D \cdot K' \frac{W}{L}} = \sqrt{2 \cdot 100 \cdot 110 \cdot 10} = 469\mu\text{S}$$

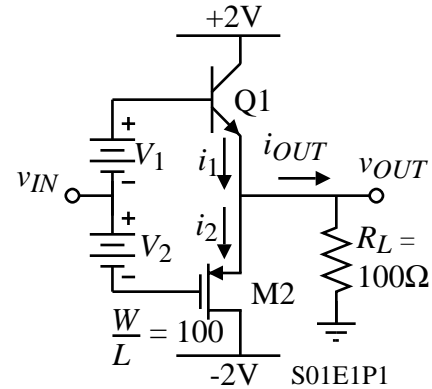
$$r_{ds} = \frac{1}{100\mu\text{A} \cdot 0.04} = 0.25\text{M}\Omega \quad \therefore \boxed{R_{out} = 20\text{k}\Omega \parallel 29.3\text{M}\Omega \approx 20\text{k}\Omega}$$

(c.) The small signal voltage gain can be expressed as

$$\boxed{\frac{v_{out}}{v_{in}} = -g_m R_{out} = -(469\mu\text{S})(0.02\text{M}\Omega) = -9.38\text{V}/\text{V}}$$

Problem 2

A push-pull follower is shown which uses an NPN BJT and a p-channel MOSFET. In this problem, ignore the bulk effect, the channel length modulation, and the Early voltage. The parameters for the NPN BJT are $\beta_F = 100$, $I_s = 10\text{fA}$ and $V_t = 25.9\text{mV}$. The model parameters for the PMOS are $K_P' = 50\mu\text{A}/\text{V}^2$ and $V_{TP} = -0.7\text{V}$. (a.) Find the value of the dc batteries, V_1 and V_2 , which will cause $100\mu\text{A}$ to flow in Q1 and M2 when the dc value of $v_{IN} = 0\text{VDC}$. (b.) Find the small-signal input resistance, output resistance (not including R_L) and voltage gain when the dc value of $v_{IN} = 0\text{VDC}$.



Solution

(a.) $V_1 = V_{BE1} = V_t \ln\left(\frac{i_C}{I_s}\right) = 0.0259 \ln\left(\frac{100\mu\text{A}}{10\text{fA}}\right) = 0.5964\text{V} \rightarrow \boxed{V_1 = 0.5964\text{V}}$

$V_2 = V_{SG2} = \sqrt{\frac{2I_D}{K_P'(W/L)}} + |V_{TP}| = \sqrt{\frac{2 \cdot 100}{50 \cdot 100}} + 0.7 = 0.9\text{V} \rightarrow \boxed{V_2 = 0.9\text{V}}$

(b.) Small-signal model (simplified):

$g_{m1} = \frac{I_{C1}}{V_t} = \frac{100\mu\text{A}}{25.9\text{mV}} = 3.86\text{mS}$

$r_{\pi 1} = \frac{1 + \beta_F}{g_{m1}} = 26.159\text{k}\Omega$

$g_{m2} = \sqrt{\frac{2K_P'W_2I_{D2}}{L_2}} = \sqrt{2 \cdot 50 \cdot 100 \cdot 100} = 1\text{mS}$

$R_{in} : v_{in} = r_{\pi 1} i_{in} + (i_{in} + g_{m1} v_{\pi} + g_{m2} v_{gs2}) R_L = r_{\pi 1} i_{in} + (i_{in} + g_{m1} r_{\pi 1} i_{in} + g_{m2} r_{\pi 1} i_{in}) R_L$

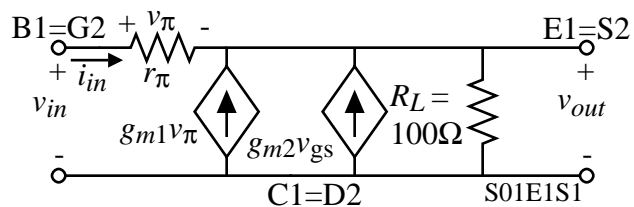
$R_{in} = \frac{v_{in}}{i_{in}} = r_{\pi 1} + R_L + g_{m1} r_{\pi 1} R_L + g_{m2} r_{\pi 1} R_L = r_{\pi 1} + R_L(1 + \beta_F) + g_{m2} r_{\pi 1} R_L$

$\therefore R_{in} = 26.159\text{k}\Omega + 101 \cdot 100\Omega + 1 \cdot 26.159\text{k}\Omega \cdot 0.1 = 38.875\text{k}\Omega \rightarrow \boxed{R_{in} = 38.875\text{k}\Omega}$

$R_{out} : R_{out} = \frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}} = \frac{1}{3.86\text{mS} + 1\text{mS}} = 205.8\Omega \rightarrow \boxed{R_{out} = 205.8\Omega}$

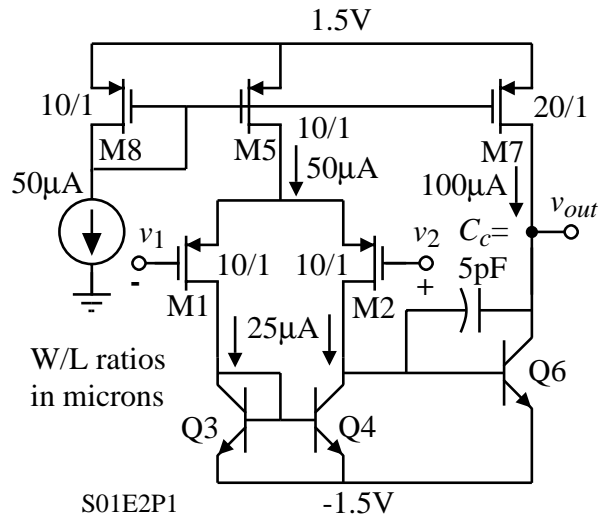
$\frac{v_{out}}{v_{in}} : \frac{v_{out}}{v_{in}} = \frac{v_{out} i_{in}}{i_{in} v_{in}} = \frac{R_L(1 + \beta_F) + g_{m2} r_{\pi 1} R_L}{r_{\pi 1} + R_L(1 + \beta_F) + g_{m2} r_{\pi 1} R_L} = \frac{12.716}{38.875} = 0.3271$

$\boxed{\frac{v_{out}}{v_{in}} = 0.3271\text{V/V}}$



Day 4 - Solutions**Problem 1**

A two-stage, BiCMOS op amp is shown. For the PMOS transistors, the model parameters are $K_P' = 50\mu\text{A}/\text{V}^2$, $V_{TP} = -0.7\text{V}$ and $\lambda_P = 0.05\text{V}^{-1}$. For the NPN BJTs, the model parameters are $\beta_F = 100$, $V_{CE}(\text{sat}) = 0.2\text{V}$, $V_A = 25\text{V}$, $V_t = 26\text{mV}$, $I_s = 10\text{fA}$ and $n=1$. (a.) Identify which input is positive and which input is negative. (b.) Find the numerical values of differential voltage gain magnitude, $|A_v(0)|$, GB (in Hertz), the slew rate, SR , and the location of the RHP zero. (c.) Find the numerical value of the maximum and minimum input common mode voltages.

**Solution**

(a.) The plus and minus signs on the schematic show which input is positive and negative.

(b.) The differential voltage gain, $A_v(0)$, is given as

$$A_v(0) = \frac{g_{m1}}{g_{ds2} + g_{o4} + g_{\pi6}} \cdot \frac{g_{m6}}{g_{ds7} + g_{o6}} \quad g_{m1} = g_{m2} = \sqrt{2 \cdot 50 \cdot 25 \cdot 10} = 158.1\mu\text{S}$$

$$r_{ds2} = \frac{1}{\lambda_P I_D} = \frac{20}{25\mu\text{A}} = 0.8\text{M}\Omega, \quad r_{o4} = \frac{V_A}{I_C} = \frac{25\text{V}}{25\mu\text{A}} = 1\text{M}\Omega, \quad g_{m6} = \frac{I_C}{V_t} = \frac{100\mu\text{A}}{26\text{mV}} = 3846\mu\text{S}$$

$$r_{\pi6} = \frac{\beta_F}{g_{m6}} = 26\text{k}\Omega, \quad r_{ds7} = \frac{1}{\lambda_P I_D} = \frac{20}{100\mu\text{A}} = 0.2\text{M}\Omega \quad \text{and} \quad r_{o6} = \frac{V_A}{I_C} = \frac{25\text{V}}{100\mu\text{A}} = 0.25\text{M}\Omega$$

$$\therefore |A_v(0)| = [158.1(0.8 || 1 || 0.026)][3846(0.2 || 0.25)] = 3.888 \cdot 427.36 = \underline{\underline{1,659.6\text{V/V}}}$$

$$GB = \frac{g_{m1}}{C_c} = \frac{158.1\mu\text{S}}{5\text{pF}} = 31.62 \times 10^6 \text{ rads/sec} \rightarrow \underline{\underline{GB = 5.0325\text{MHz}}}$$

$$SR = \frac{50\mu\text{A}}{5\text{pF}} = \underline{\underline{10\text{V}/\mu\text{s}}}$$

$$\text{RHP zero} = \frac{g_{m6}}{C_c} = \frac{3.846\text{mS}}{5\text{pF}} = \underline{\underline{769.24 \times 10^6 \text{ rads/sec. (122MHz)}}}$$

(c.) The maximum input common mode voltage is given as

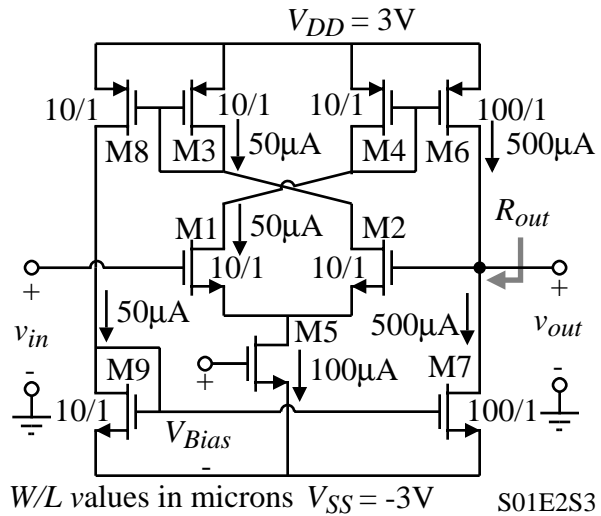
$$v_{icm}^+ = V_{CC} - V_{DS5}(\text{sat}) - V_{SG1} = 1.5 - \sqrt{\frac{2 \cdot 50}{50 \cdot 10}} - 0.7 - \sqrt{\frac{2 \cdot 25}{50 \cdot 10}} = 0.8 - 0.447 - 0.316 =$$

$$\therefore v_{icm}^+ = \underline{\underline{0.0367\text{V}}}$$

$$v_{icm}^- = -1.5 + V_{BE3} - V_{T1} = -1.5 + V_t \ln\left(\frac{25\mu\text{A}}{10\text{fA}}\right) - 0.7 = -2.2 + 0.5626 = \underline{\underline{-1.6374\text{V}}}$$

Problem 2

A CMOS circuit used as an output buffer for an OTA is shown. Find the value of the small signal output resistance, R_{out} , and from this value estimate the -3dB bandwidth if a 50pF capacitor is attached to the output. What is the maximum and minimum output voltage if a 1k Ω resistor is attached to the output? What is the quiescent power dissipation of this circuit? Use the following model parameters: $K_N' = 110\mu\text{A}/\text{V}^2$, $K_P' = 50\mu\text{A}/\text{V}^2$, $V_{TN} = -V_{TP} = 0.7\text{V}$, $\lambda_N = 0.04\text{V}^{-1}$ and $\lambda_P = 0.05\text{V}^{-1}$.



Solution

Use feedback concepts to calculate the output resistance, R_{out} .

$$R_{out} = \frac{R_o}{1-LG}$$

where R_o is the output resistance with the feedback open and LG is the loop gain.

$$R_o = \frac{1}{g_{ds6} + g_{ds7}} = \frac{1}{(\lambda_N + \lambda_P)I_6} = \frac{10^6}{0.09 \cdot 500} = 22.22\text{k}\Omega$$

The loop gain is,

$$LG = \frac{v_{out}'}{v_{out}} = -\frac{1}{2} \left[\frac{g_{m2}g_{m6}}{g_{m4}} + \frac{g_{m1}g_{m9}}{g_{m7}} \right] R_o$$

$$g_{m1} = g_{m2} = \sqrt{2 \cdot 110 \cdot 50 \cdot 10} = 331.67\mu\text{S}, \quad g_{m3} = g_{m4} = \sqrt{2 \cdot 50 \cdot 50 \cdot 10} = 223.6\mu\text{S},$$

$$g_{m6} = \sqrt{2 \cdot 50 \cdot 100 \cdot 500} = 2236\mu\text{S} \quad \text{and} \quad g_{m7} = \sqrt{2 \cdot 110 \cdot 500 \cdot 100} = 3316.7\mu\text{S}$$

$$\therefore LG = \frac{v_{out}'}{v_{out}} = -\frac{1}{2} \left[\frac{-331.67 \cdot 2236}{223.6} + \frac{-331.67 \cdot 3316.7}{331.67} \right] = -73.68\text{V/V}$$

$$R_{out} = \frac{R_o}{1-LG} = \frac{22.22\text{k}\Omega}{1+73.68} = \underline{\underline{294.5\Omega}}$$

$$f_{-3\text{dB}} = \frac{1}{2\pi \cdot R_{out} \cdot 50\text{pF}} = \frac{1}{2\pi \cdot 294.5 \cdot 50\text{pF}} = \underline{\underline{10.81\text{MHz}}}$$

To get the maximum swing, we must check two limits. First, the saturation voltages of M6 and M7.

$$V_{ds6(\text{sat})} = \sqrt{\frac{2 \cdot 1000}{50 \cdot 100}} = 0.6325\text{V} \quad \text{and} \quad V_{ds7(\text{sat})} = \sqrt{\frac{2 \cdot 1000}{110 \cdot 100}} = 0.4264\text{V}$$

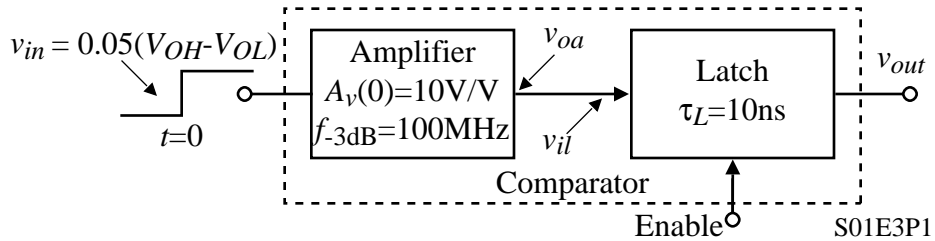
Second, the maximum current available to the 1k Ω resistor is $\pm 1\text{mA}$ which means that the output swing can only be $\pm 1\text{V}$. Therefore, maximum/minimum output = $\pm 1\text{V}$.

$$P_{diss} = 6\text{V}(650\mu\text{A}) = \underline{\underline{3.9\text{mW}}}$$

Day 5 - Solutions

Problem 1

A comparator consists of an amplifier cascaded with a latch as shown below. The amplifier has a voltage gain of 10V/V and $f_{-3dB} = 100\text{MHz}$ and the latch has a time constant of 10ns. The maximum and minimum voltage swings of the amplifier and latch are V_{OH} and V_{OL} . When should the latch be enabled after the application of a step input to the amplifier of $0.05(V_{OH}-V_{OL})$ to get minimum overall propagation time delay? What is the value of the minimum propagation time delay? It may be useful to recall that the propagation time delay of the latch is given as $t_p = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right)$ where v_{il} is the latch input (ΔV_i of the text).



Solution

The solution is based on the figure shown. We note that,

$$v_{oa}(t) = 10[1 - e^{-\omega_{3dB}t}]0.05(V_{OH} - V_{OL}).$$

If we define the input voltage to the latch as,

$$v_{il} = x \cdot (V_{OH} - V_{OL})$$

then we can solve for t_1 and t_2 as follows:

$$x \cdot (V_{OH} - V_{OL}) = 10[1 - e^{-\omega_{3dB}t_1}]0.05(V_{OH} - V_{OL}) \rightarrow x = 0.5[1 - e^{-\omega_{3dB}t_1}]$$

This gives,

$$t_1 = \frac{1}{\omega_{3dB}} \ln\left(\frac{1}{1-2x}\right)$$

From the propagation time delay of the latch we get,

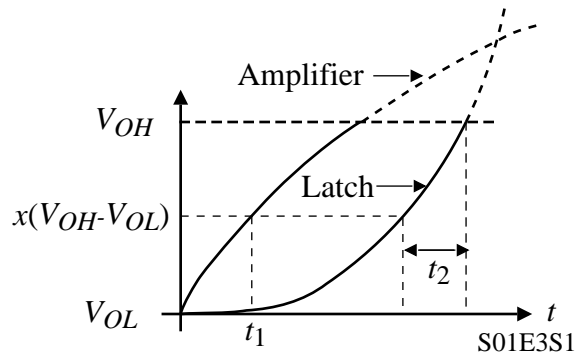
$$t_2 = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right) = \tau_L \ln\left(\frac{1}{2x}\right)$$

$$\therefore t_p = t_1 + t_2 = \frac{1}{\omega_{3dB}} \ln\left(\frac{1}{1-2x}\right) + \tau_L \ln\left(\frac{1}{2x}\right) \rightarrow \frac{dt_p}{dx} = 0 \text{ gives } x = \frac{\pi}{1+2\pi} = 0.4313$$

$$t_1 = \frac{10\text{ns}}{2\pi} \ln(1+2\pi) = 1.592\text{ns} \cdot 1.9856 = \underline{3.16\text{ns}} \text{ and } t_2 = 10\text{ns} \ln\left(\frac{1+2\pi}{2\pi}\right) =$$

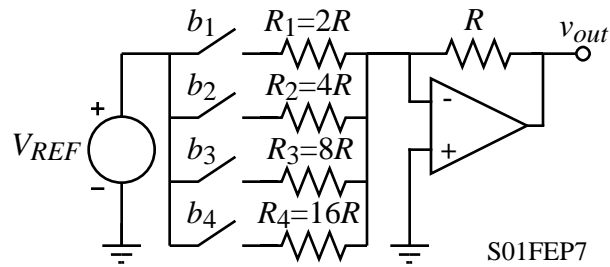
$$1.477\text{ns}$$

$$\therefore t_p = t_1 + t_2 = 3.16\text{ns} + 1.477\text{ns} = \underline{4.637\text{ns}}$$



Problem 2

A 4-bit, binary-weighted, resistor DAC is shown. (a.) Assume that $GB = \infty$ and find the minimum differential gain, $A_{vd}(0)$, required for the DAC to have an INL of $\pm 0.5LSB$. (b.) If the differential gain is very large, find the conversion time of this DAC if $GB = 1MHz$.

Solution

(a.) First we need to find $v_{out}(\text{ideal})$ and $v_{out}(\text{actual})$ for the worst case condition which is when all switches are closed (closed loop gain is highest).

$$v_{out}(\text{ideal}) = -\frac{R_{EQ}}{R} V_{REF} \quad \text{and} \quad v_{out}(\text{actual}) = -\left(\frac{\frac{AR}{R+R_{EQ}}}{1 + \frac{AR_{EQ}}{R+R_{EQ}}} \right) V_{REF}$$

$$\text{where } R_{EQ} = \frac{R}{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}} = \frac{32R}{30}$$

$$\text{Now, } v_{out}(\text{actual}) - v_{out}(\text{ideal}) \leq \frac{V_{REF}}{32} \Rightarrow -\left(\frac{\frac{AR}{R+R_{EQ}}}{1 + \frac{AR_{EQ}}{R+R_{EQ}}} \right) V_{REF} + \frac{R_{EQ}}{R} V_{REF} \leq \frac{V_{REF}}{32}$$

$$\text{Which simplifies to, } \frac{32R}{R_{EQ}} - 1 \leq \frac{AR_{EQ}}{R+R_{EQ}} \Rightarrow A \geq \frac{R+R_{EQ}}{R_{EQ}} \left(\frac{32R}{R_{EQ}} - 1 \right)$$

$$A \geq \frac{15R+16R}{16R} \left(\left(\frac{32R}{32R} \right) 30 - 1 \right) = \frac{31}{16} 29 = \underline{\underline{56.2V/V}}$$

$$(b.) \text{ In this case, } v_{out}(\text{ideal}) = -\frac{30}{32} V_{REF} \quad \text{and} \quad v_{out}(\text{actual}) = -\frac{30}{32} [1 - e^{-t\omega_H}] V_{REF}$$

$$\text{where } \omega_H = \frac{GB \cdot R_{EQ}}{R+R_{EQ}} = \frac{2\pi \times 10^6 (32/30)}{1 + (32/30)} = (32/31)\pi \times 10^6 \text{ rads/sec.}$$

$$\text{Now, } v_{out}(\text{actual}) - v_{out}(\text{ideal}) \leq \frac{V_{REF}}{32} \Rightarrow -\frac{30}{32} [1 - e^{-T\omega_H}] V_{REF} + \frac{30}{32} V_{REF} \leq \frac{V_{REF}}{32}$$

$$\text{which becomes, } 30 e^{-\omega_H T} = 1 \Rightarrow T = \frac{1}{\omega_H} \ln(30) = \frac{31}{32\pi} \ln(30) \mu\text{s}$$

$$\therefore \underline{\underline{T = 1.0488\mu\text{s}}}$$