LECTURE 070 –MODULATION AND DEMODULATION USING PLLS

INTRODUCTION

Objective

The objective of this presentation is:

1.) Show the applications of a PLL for modulation and demodulation at the system level

2.) Introduce the concepts of phase noise and spurious responses

Outline

- Review of Modulation
- Phase Noise
- Use of a PLL for Modulation and Demodulation
- Frequency Synthesizers
- Continuation of the Design of a 450-475 MHz DPLL Synthesizer
- Summary

REVIEW OF MODULATION

Amplitude Modulation

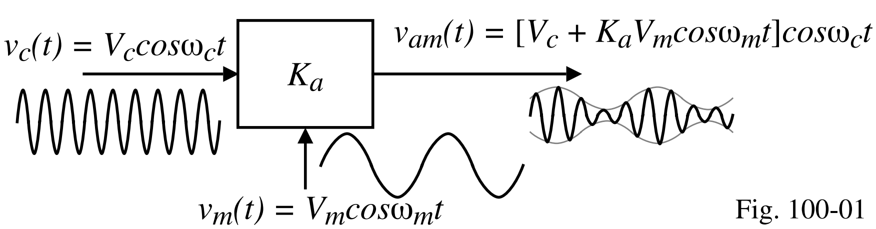


Fig. 100-01

$$v_{am}(t) = V_c[1 + m_a\, cos\omega_m t]\, cos\omega_c t$$

where

$$m_a = \text{modulation index} = \frac{K_a V_m}{V_c}$$

$$v_{am}(t) = V_c cos\omega_c t + m_a V_c\, cos\omega_m t\, cos\omega_c t$$

$$= V_c cos\omega_c t + V_c[\frac{m_a}{2} cos(\omega_c t+\omega_m t) + \frac{m_a}{2} cos(\omega_c t-\omega_m t)]$$

Spectrally,

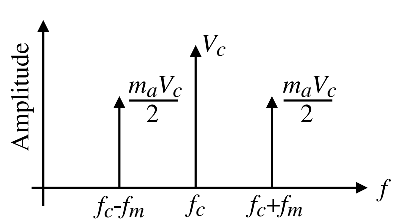


Fig. 100-02

Spurs Caused by Unintentional AM

An example:

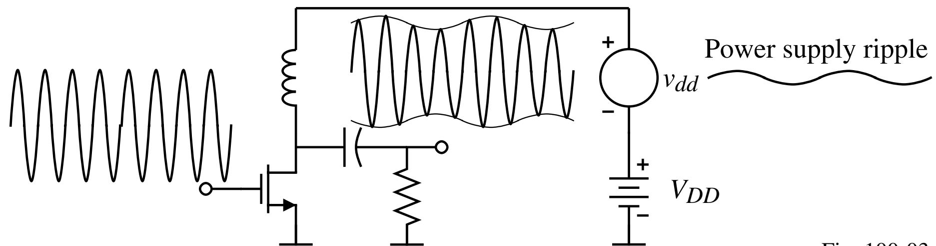


Fig. 100-03

In a frequency synthesizer, AM modulation is unwanted and m_a<<1.

The single-sideband (SSB) to carrier ratio is given as,

$$\text{Power of carrier} = P_c = \frac{(V_c)^2}{R} \qquad \text{and} \qquad \text{Power of sideband} = P_{side} = \frac{\left(m_a \frac{V_c}{2}\right)^2}{R}$$

$\therefore$ The SSB spur to carrier ratio in dBc is given as

$$20\log_{10}\left(\frac{P_{side}}{P_c}\right) = 20\log_{10}\left(\frac{m_a}{2}\right) \text{ dBc}$$

Thus a small amplitude modulation index can cause reasonably large spurs.

Removal of AM by Amplitude Limiting

Amplitude limiting can be used to remove AM.

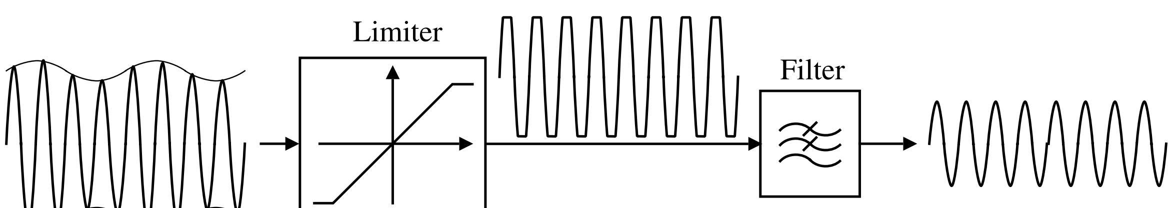


Fig. 100-04

It will turn out that phase noise has two components – amplitude noise and phase noise. Because of the above example, phase noise is much more important.

Illustration of amplitude and phase noise:

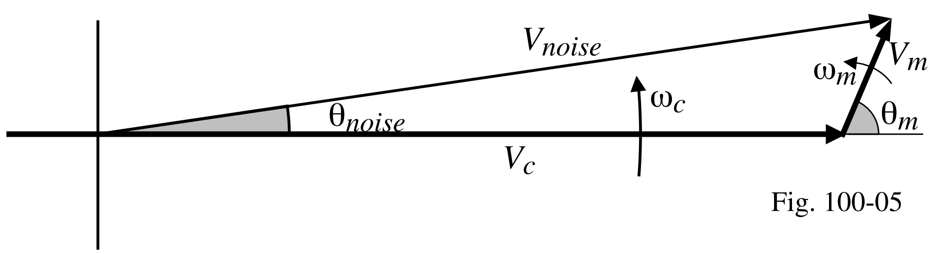


Fig. 100-05

Frequency Modulation

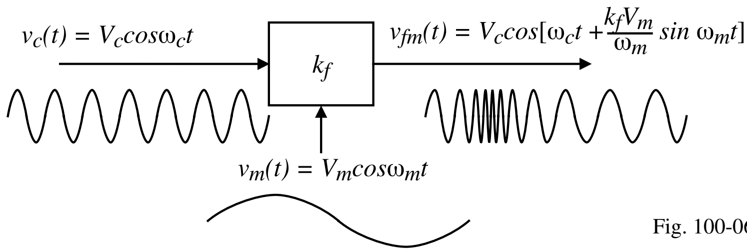


Fig. 100-06

Output frequency:

$$\omega_o(t) = \omega_c + k_f V_m cos\ \omega_m t = \omega_c + \Delta\omega_c cos\ \omega_m t$$

The peak value of $\omega_c = \Delta\omega_c = k_f V_m$ (called the frequency deviation)

$$\theta(t) = \int \omega_o(t)\mathrm{dt} = \int[\omega_c + k_f V_m cos\ \omega_m t]\mathrm{dt} = \omega_c t + \frac{k_f V_m}{\omega_m} sin\ \omega_m t$$

$$\therefore \quad v_{fm}(t) = V_c \cos\left(\omega_c t + \frac{k_f V_m}{\omega_m} sin\ \omega_m t\right) = V_c \cos(\omega_c t + \beta\ sin\ \omega_m t)$$

where

$$\beta = \text{modulation index} = \frac{\Delta\omega_c(\text{peak})}{\omega_m} = \frac{k_f V_m}{\omega_m}$$

Phase Modulation

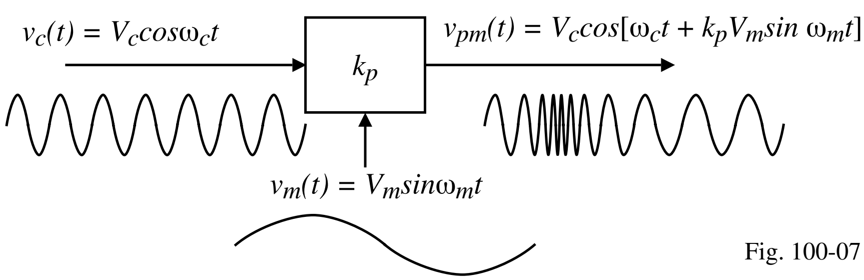


Fig. 100-07

Peak phase deviation and modulation index = $\theta_d = k_p V_m$

$$v_{pm}(t) = V_c \cos(\omega_c t + k_p V_m\ sin\ \omega_m t) = V_c \cos(\omega_c t + \theta_d\ sin\ \omega_m t)$$

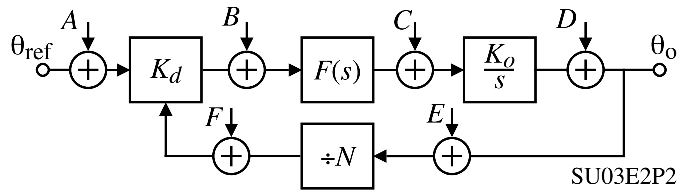
Note that in the time domain, FM and PM are identical.

$$v_{fm}(t) = V_c \cos(\omega_c t + \beta\ sin\ \omega_m t)$$

$$v_{pm}(t) = V_c \cos(\omega_c t + \theta_d\ sin\ \omega_m t)$$

Example 1 – Phase Modulation of a PLL

The frequency synthesizer shown has the following parameters:



$$F(s) = \frac{1+0.01s}{s} \qquad K_o = 2\text{x}10^6 \text{ (rads/V)}$$

$$K_d = 0.8 \text{ (V/rad.)} \quad \beta = 2\pi \quad N = 150$$

$$f_{ref} = 120 \text{ kHz}$$

(a.) Where would you introduce the modulating voltage, v_p, if you wish to phase modulate the output of the synthesizer (A, B, C, D, E, or F)?

(b.) What is the peak amplitude of a 1kHz ac signal needed to produce an output peak phase deviation of 0.5 radians?

Solution

(a.) The modulating voltage should be introduced at B.

(b.) The transfer function between the input modulating voltage and the output phase is given as,

$$\theta_o(s) = \frac{K_o}{s} F(s)\left[V_p(s) - K_d \frac{\theta_o}{N}\right] \quad \rightarrow \quad \theta_o(s)\left[1 + \frac{K_oK_dF(s)}{sN}\right] = \frac{K_oF(s)}{s} V_p(s)$$

Example 1 - Continued

$$\frac{\theta_o(s)}{V_p(s)} = \frac{K_oF(s)}{s + \frac{K_vF(s)}{N}} = \frac{K_o(1+0.01s)}{s^2 + \frac{0.01K_v}{N}s + \frac{K_v}{N}}$$

$$\therefore \omega_n = \sqrt{\frac{K_v}{N}} = \sqrt{\frac{1.6\text{x}10^6}{150}} = 103 \text{ rads/sec. (16.4 Hz) and } \zeta = \frac{K_v}{100N}\sqrt{\frac{N}{K_v}} \approx 1$$

Since, f_n << 1kHz, the transfer function can be approximated as,

$$\left|\frac{\theta_o(j\omega)}{V_p(j\omega)}\right| \approx \frac{0.01K_o}{\omega} = \frac{20{,}000}{2000\pi} = 3.183$$

$\therefore$ A phase deviation of 0.5 radians requires a modulating voltage of 0.5/3.183 or 0.157V

Peak deviation of the modulating voltage = 0.157V

Example 2 – Phase Modulation of a DPLL

A DPLL frequency synthesizer has the following parameters:

$$F(s) = \frac{1+0.01s}{s} \qquad K_o = 2\text{x}10^6 \text{ (rads/V)} \qquad K_d = 0.8\text{V/rad}$$

$$\beta = 2\pi \qquad N = 150 \qquad f_{ref} = 120 \text{ kHz}$$

The temperature is 290°K and all circuits operate from a ±5V power supply.

(a.) What is the output frequency hold range in Hz?

(b.) What is the output frequency lock (capture) range in Hz? What is the lock (capture) time in seconds?

(c.) Assume that you wish to phase modulate the output of the synthesizer, where would you introduce the modulating voltage? What is the peak amplitude of the 1 kHz ac signal needed to produce an output peak phase deviation of 0.5 radians?

Solution

(a.) Since the filter is active PI, the hold range is limited by the loop components and is

$$\Delta\omega_{max} = \Delta\omega_H = \pm 5\text{V}\cdot K_o = \pm 5\text{V}(2\text{x}10^6 \text{ (rads/V)} = \pm 10\text{Mrads/sec.}$$

$$\therefore \ \Delta f_H = \frac{\pm 10\text{Mrads/sec}}{2\pi} = \pm 1.592 \text{ MHz} \ \rightarrow \ \boxed{\Delta f_H = \pm 1.592 \text{ MHz}}$$

(b.) First we find K (Lecture 90-02).

$$K = \frac{K_d K_o}{N} = \frac{0.8\cdot 2\text{x}10^6}{150} = 10{,}667$$

Example 2 – Continued

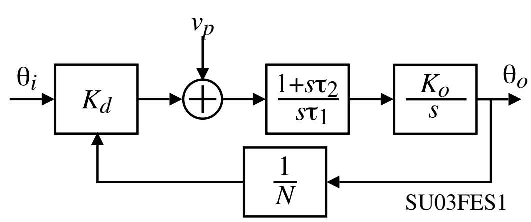
From the lecture notes,

$$\omega_n = \sqrt{\frac{K}{\tau_1}} = \sqrt{\frac{10{,}667}{1}} = 103.3 \text{ Rads/sec} \quad \text{and} \quad \zeta = \frac{\tau_2\omega_n}{2} = \frac{0.01\cdot 103.3}{2} = 0.516$$

$$\therefore \quad \Delta f_L = \frac{2\beta N\zeta\omega_n}{2\pi} = \frac{2(2\pi)150\cdot 0.516\cdot 103.3}{2\pi} = 15{,}991 \text{ Hz} \ \rightarrow \ \boxed{\Delta f_L = \pm 15{,}991\text{Hz}}$$

$$t_L = \frac{2\pi}{\omega_n} = \frac{2\pi}{103.3} = 60.8 \text{ msec} \qquad \boxed{t_L = 60.8 \text{ msec}}$$

(c.) See the following block diagram.



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What is the BW? From the lecture notes,

$$BW = 103.3\sqrt{2(0.516^2)+1+\sqrt{2(0.516^2)+1}}$$

$$= 103.3(1.546) = 159.75 \text{ rads/sec.}$$

or $BW = 25.42$ Hz

Since, 1kHz >> BW, we can write,

$$\frac{\theta_o}{v_p} \approx \frac{\tau_2 K_o}{\tau_1 \omega} \ \rightarrow \ v_p = \theta_o\frac{\tau_1\omega}{\tau_2 K_o} = 0.5\left(\frac{1\cdot 2000\pi}{0.01\cdot 2\text{x}10^6}\right) = 0.157\text{V} \qquad \boxed{v_p = 0.157\text{V}}$$

Spectrum of FM Modulation

Find the frequency domain equivalence of FM modulation:

Using Bessel function of the first kind with order n, we get

$$v_{fm}(t) = V_c \{J_0(\beta)\sin\omega_c t + J_1(\beta)[\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t]$$
$$+ J_2(\beta)[\sin(\omega_c + 2\omega_m)t - \sin(\omega_c - 2\omega_m)t] + J_3(\beta)[\sin(\omega_c + 3\omega_m)t - \sin(\omega_c - 3\omega_m)t] + \cdots\}$$

Spectrum:

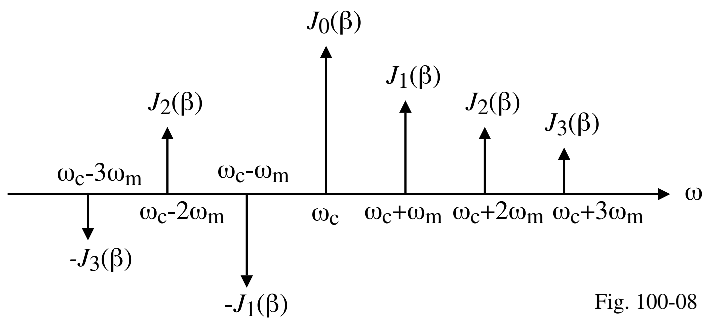


Fig. 100-08

Observations:

- The modulation index, β, controls the number of sidebands
- The spacing between the sidebands is f_m
- $BW \approx 2(\Delta f_{peak} + f_m)$ (Carson's rule)

For narrowband FM, $\beta << 1$

$\therefore \quad J_0(\beta) \approx 1, \; J_1(\beta) \approx 0.5\beta, \;$ and $\; J_n(\beta) \approx 0$ if $n \geq 2$

FM and PM Spurs ($\beta << 1$)

Spurs are unintentional and not wanted.

If $\beta << 1$ and $\theta_d << 1$, then $\beta \approx \theta_d$

The spectrum for FM or PM becomes,

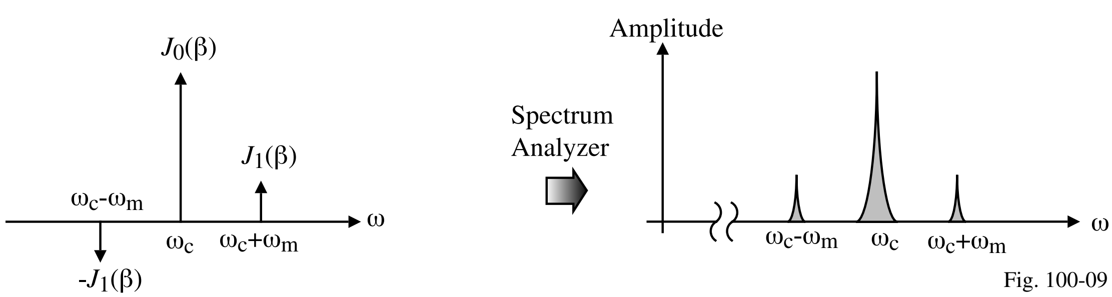


Fig. 100-09

SSB spur to carrier ratio = $20\,log_{10}\left(\frac{\beta}{2}\right)$ for FM

SSB spur to carrier ratio = $20\,log_{10}\left(\frac{\theta_d}{2}\right)$ for PM

In general the SSB spur to carrier ratio of AM, FM or PM is

$$20\,log_{10}\left(\frac{\text{Modulation Index}}{2}\right) \text{ dBc}$$

SSB Spurs

Example:

Find the SSB spur if a VCO power supply has a ripple of 10μV(peak) at 1000Hz that is superimposed on the control voltage of the VCO.

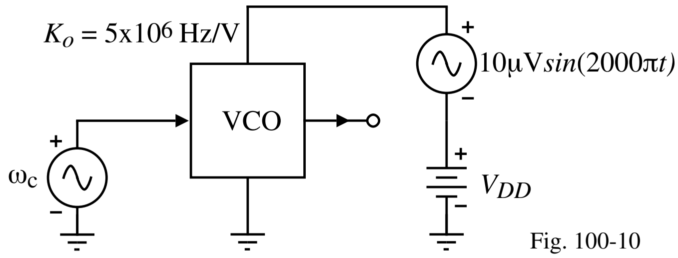


Fig. 100-10

Assuming that the power supply ripple is superimposed on the controlling voltage, we get,

$$\Delta f_c = (5\text{x}10^6 \text{ Hz/V})(10\mu\text{V}) = 50 \text{ Hz}$$

The unintentional modulation frequency is 1000Hz.

$$\therefore \quad \beta = \frac{\Delta f_c}{f_m} = \frac{50}{1000} = 0.050 \quad \rightarrow \quad \text{SSB Spur} = 20\ log_{10}\left(\frac{0.05}{2}\right) = \text{-34 dBc}$$

Influence of Frequency Multiplication on Spurs

Consider the case of a frequency doubler.

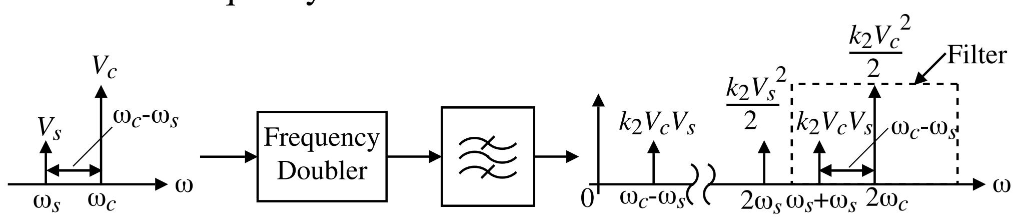


Fig. 100-11

$$v_o(t) = k_2(V_c \cos\omega_c t + V_s \cos\omega_s t)^2 = k_2(V_c^2\cos^2\omega_c t + V_s^2\cos^2\omega_s t + 2V_cV_s \cos\omega_c t \cos\omega_s t)$$

$$= k_2\frac{V_c^2}{2}(1 + \cos 2\omega_c t) + k_2\frac{V_s^2}{2}(1 + \cos 2\omega_s t) + k_2V_cV_s[\cos(\omega_c+\omega_s)t + \cos(\omega_c-\omega_s)t]$$

Desired output is $k_2\frac{V_c^2}{2}\cos 2\omega_c t$

The inband spur is $k_2V_cV_s\cos(\omega_c+\omega_s)t$

$$\therefore \quad \frac{\text{Power of spur}}{\text{Power of carrier}} = \frac{(k_2V_cV_s)^2}{\left(k_2\frac{V_c^2}{2}\right)^2} = 4\left(\frac{V_s}{V_c}\right)^2 \text{ at the output } \rightarrow \text{SSB} = 20\log_{10}\left(\frac{V_s}{V_c}\right) + 6.02\text{dB}$$

The spur-to-carrier ratio at the input is $\left(\frac{V_s}{V_c}\right)^2 \rightarrow \text{SSB} = 20\log_{10}\left(\frac{V_s}{V_c}\right)$

In general for a ×n multiplier we see that SSB(output) = SSB(input) + 20log(n)

Effect of Frequency Multiplication on FM/PM Spurs

Let,

$$\omega = \omega_c + \Delta\omega_c(\text{peak}) \cos\omega_m t$$

For spurs,

$$\beta = \frac{\Delta\omega_c(\text{peak})}{\omega_m} << 1 \quad \Rightarrow \quad \beta \approx \theta_d$$

Multiplying by n gives the new frequency as

$$\omega_{new} = n\omega = n\omega_c + n\Delta\omega_c(\text{peak}) \cos\omega_m t$$

$$\beta_{new} = \frac{n\Delta\omega_c(\text{peak})}{\omega_m} = n\beta$$

Thus,

$$\text{SSB} = 20\log_{10}\left(\frac{\beta_{new}}{n}\right) + 20\log_{10}(n) \quad (\text{FM})$$

$$\text{SSB} = 20\log_{10}\left(\frac{\theta_{d,new}}{n}\right) + 20\log_{10}(n) \qquad (\text{PM})$$

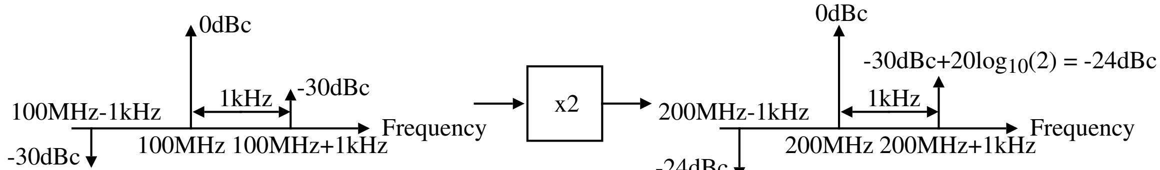


Fig. 100-12

Effect of Frequency Multiplication on FM/PM Spurs – Continued

From the previous results, we see that as n increases, the spur level at the output increases.

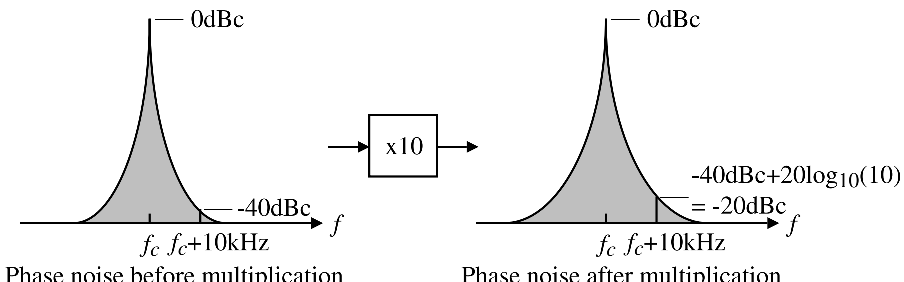


Fig. 100-13

Summary:

- Frequency multiplication increases the relative level of isolated spurs by $20\log_{10}(N)$
- The offset frequency of the spur is not affected
- Frequency multiplication does not affect the level or frequency of AM spurs
- Frequency multiplication increases the relative level of PM/FM spurs by $20\log_{10}(N)$
- The offset (modulation) frequency of the spurs is not affected
- Phase noise is affected in the same way as sinusoidal phase modulation

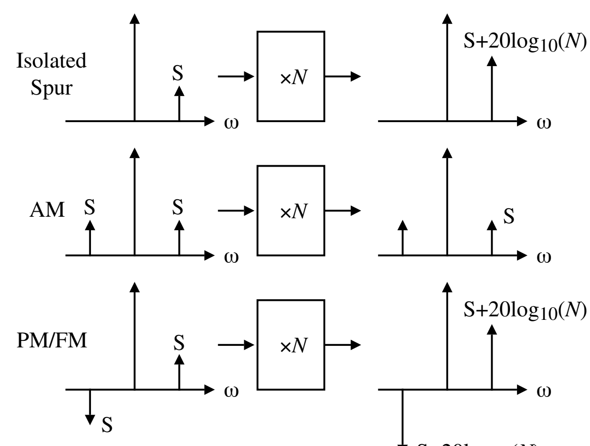


Fig. 100-135

Effect of Frequency Division on FM/PM Spurs

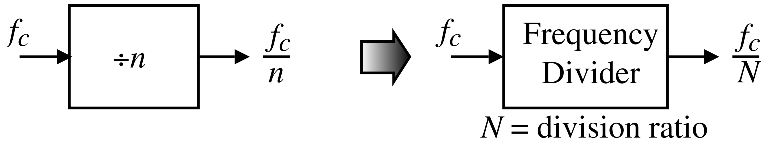


Fig. 100-14

$$SSB = SSB(\text{input}) - 20\log_{10}(N)$$

Therefore, the use of a frequency divider decreases the phase noise.

Summary:

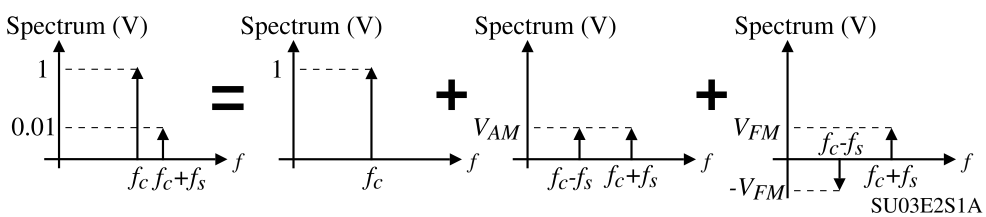
- Frequency division by N is equivalent to frequency multiplication by $1/N$
- Note that division in the feedback path is equivalent to multiplication in the forward path.

Example 3 – Influence of a Limiter on Spurs

A carrier together with a single –40dBc spur 1kHz above the carrier are applied to an ideal limiter. Sketch the spectrum of the output of the limiter considering frequencies through the 5th harmonic of the carrier. Assume the limiter acts like a square wave modulator resulting in odd harmonics only whose amplitudes are given as $a_n = (4/n\pi)$ where n = the harmonic.

Solution

The amplitude limiter will only influence the amplitude modulation so that we must resolve this spectrum into a pair of AM and FM sidebands. This will be done as follows where f_s = 1 kHz.



The amplitude of the single spur is found as

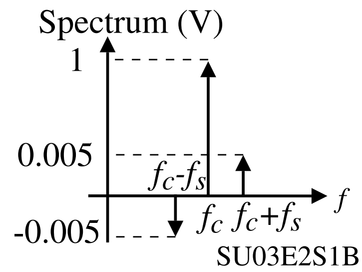
$$V_{spur} = 10^{-40/20} = 0.01\text{V}$$

We know that $V_{AM} = V_{PM}$ and that $V_{spur} = 0.01\text{V} = 2V_{AM} = 2V_{PM}$. Therefore,

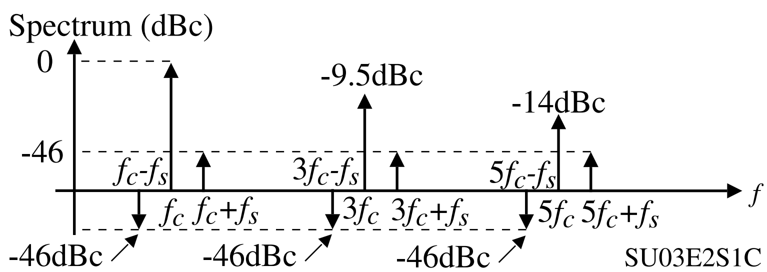
$$V_{AM} = V_{PM} = 0.005 \rightarrow \quad V_{PM}(\text{dBc}) = 20\log_{10}(0.005) = -46\text{ dBc}$$

Example 3 - Continued

The limiter removes the AM and the spectrum looks like the spectrum shown to the right.



The key to answering this question is to realize that the carrier will be reduced by *n* the harmonic number because of the mixing action of the limiter. The FM spurs will also be reduced by the same amount, but because of the multiplication, the spurs will be increased by the same amount. Thus, the spurs do not change for the various harmonics. The resulting spectrum out to the fifth harmonic is given below.



PHASE NOISE

Characteristics of Phase Noise

Phase noise is the inherent uncertainty in the instantaneous frequency of a periodic signal.

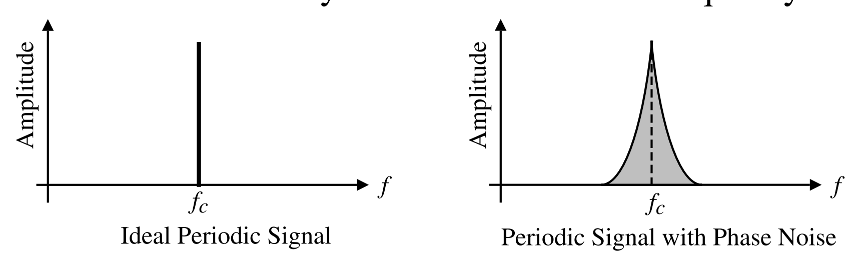


Fig. 100-01

Comments:

- Phase noise has both magnitude and phase, however, the phase component is more important
- Phase noise comes from inherent noise in the various circuits and other random fluctuations
- Phase noise is expressed in decibels with respect to the carrier (dBc)

Single Sideband Noise Spectral Density

Assume the carrier experiences a single-frequency FM or PM.

Consider the power in the sideband at an offset of f_m from the carrier in a 1Hz bandwidth:

$$\mathcal{L}(f_m) = \frac{\text{Noise power per Hz bandwidth}}{\text{Total carrier power}} = \frac{\int_{f_m+0.5}^{f_m-0.5} P_\theta(f)df}{P_c}$$

where $P_\theta(f)$ is the normalized power spectral density (W/Hz) and P_c is the total power under the power spectrum and is called the carrier power.

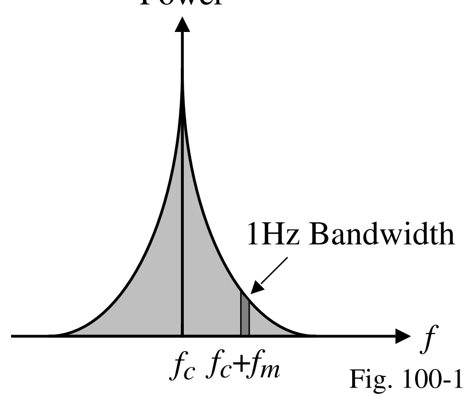


Fig. 100-15

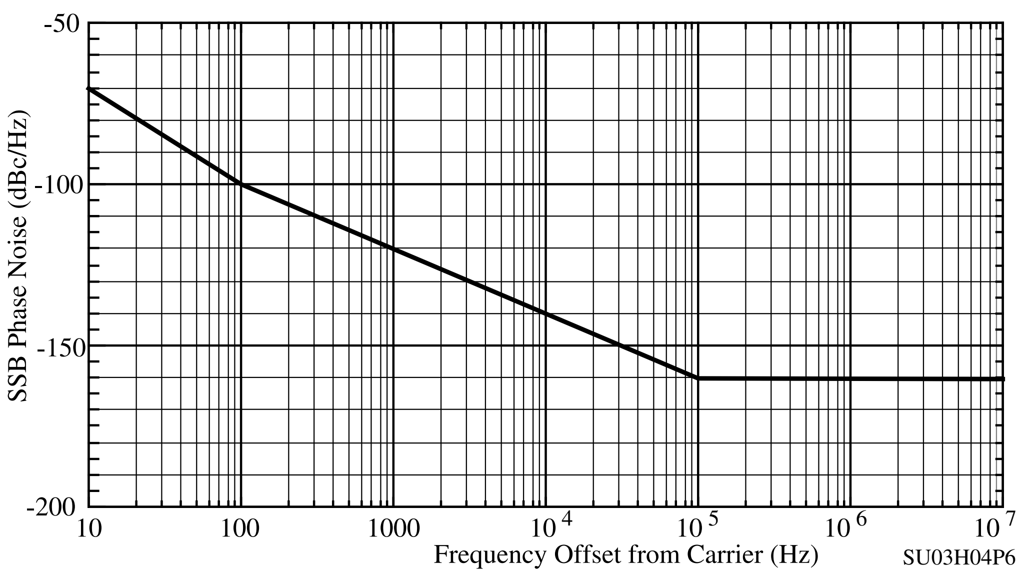
If the power spectral density is a constant over the 1 Hz bandwidth, then the phase noise can be attributed to an equivalent sine wave modulation of phase deviation, θ_d.

$$\therefore \quad \mathcal{L}(f_m) = \frac{P_\theta(f_m)}{P_c} = \frac{\left(\frac{\theta_d}{2}\right)^2}{P_c}$$

The total noise power in both sidebands is $S_\theta(f_m) = 2\mathcal{L}(f_m)$

Example of Phase Noise

Below is the phase noise data for a 1mW, 40MHz VCO.



An empirical expression for this SSB phase noise is

$$\mathcal{L}(f_m) = 10\log_{10}\left\{\frac{2FkT}{P_c}\left[1+\left(K_1\frac{f_1}{f_m}\right)\right]\left[1+\left(K_2\frac{f_2}{f_m}\right)^2\right]\right\}$$

where F, K_1 and K_2 are scaling factors and $f_1 = 200\pi$ and $f_2 = 2\pi 10^5$ in the above graph.

Reference Phase Noise

How is noise on the reference signal processed by the frequency synthesizer?

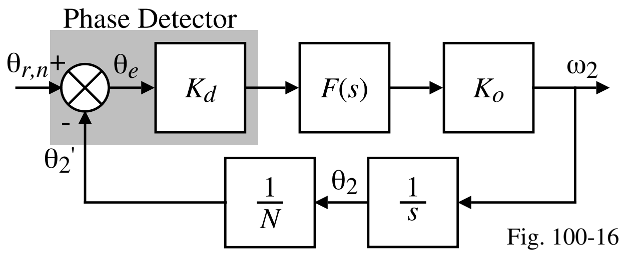


Fig. 100-16

The closed loop transfer function from the reference phase noise, $\theta_{r,n}$ is the same as for the reference phase, θ_r or θ_1, to the output phase.

$$\therefore \quad \frac{\theta_2'(s)}{\theta_{r,n}(s)} = \frac{\frac{K_vF(s)}{N}}{s + \frac{K_vF(s)}{N}}$$

(Can divide this by N to get the phase noise at the input)

Note that the PLL loop is a lowpass filter for the reference noise.

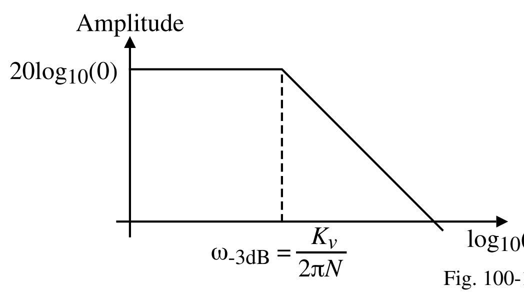


Fig. 100-17

VCO Phase Noise

How is noise on due to the VCO processed by the frequency synthesizer?

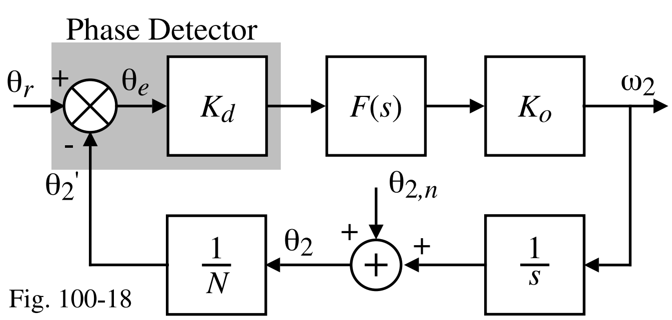


Fig. 100-18

The transfer function from the VCO noise, $\theta_{o,n}$ to the output, θ_2, is given as

$$\frac{\theta_2'(s)}{\theta_{2,n}(s)} = \frac{\frac{s}{N}}{s + \frac{K_vF(s)}{N}}$$

The PLL loop acts like a highpass filter to the VCO noise. Note that the choice of $F(s)$ does not alter the highpass nature of the relationship.

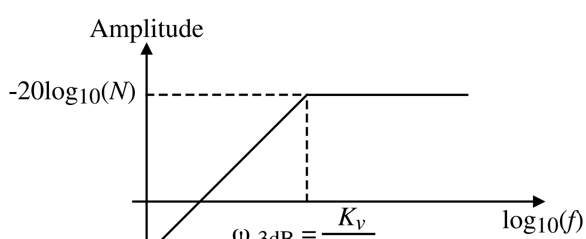


Fig. 100-19

Phase Noise at the Output Frequency

What is the phase noise at the output due to the reference noise and the VCO noise?

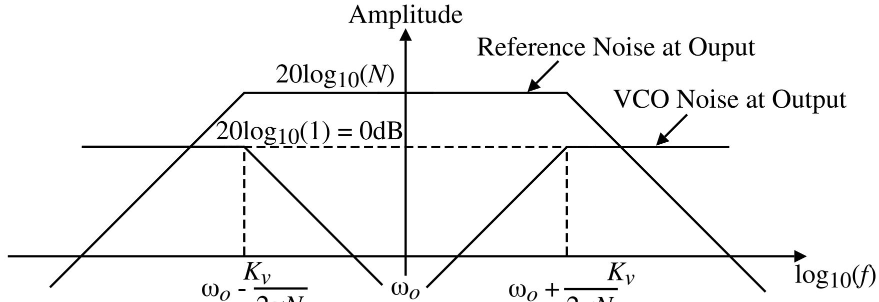


Fig. 100-20

- Note that the PLL loop acts like a bandpass filter for the reference noise and a band-reject filter for the VCO noise.
- The total noise is the rms sum of the two noises.

$$\theta_{n,total} = 20\log_{10}\left[\sqrt{\left(10^{\theta_{r,n}/20}\right)^2 + \left(10^{\theta_{VCO,n}/20}\right)^2}\right] = 10\log_{10}\left[10^{\theta_{r,n}/10} + 10^{\theta_{VCO,n}/10}\right]$$

- The total noise can be obtained by directly adding the noise powers.
- The loop bandwidth can be set to minimize the total phase noise.

USING A PLL FOR MODULATION AND DEMODULATION

FM Modulation of a PLL

A PLL is frequency modulated by introducing a baseband voltage signal, v_{fm}, into the input of the VCO.

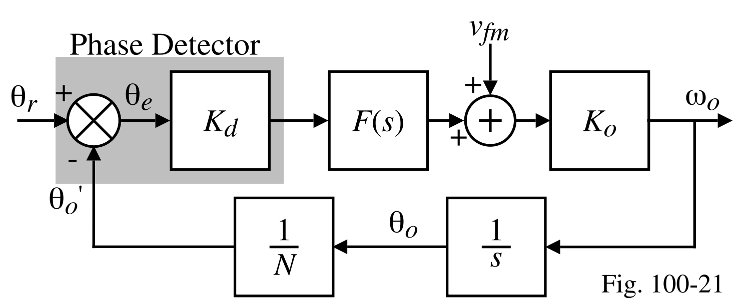


Fig. 100-21

The transfer function for FM modulation of the above PLL is,

$$\frac{\omega_o(s)}{V_{fm}(s)} = \frac{sK_o}{s + \frac{K_v F(s)}{N}}$$

The fundamental behavior of the loop is determined by letting $F(s) = 1$ to get

$$\frac{\omega_o(s)}{V_{fm}(s)} = \frac{sK_o}{s + \frac{K_v}{N}} \quad \rightarrow \quad \text{Highpass filter with a gain of } K_o \text{ and } \omega_{-3dB} = \frac{K_v}{N}$$

Note that modulation frequencies less than ω_{-3dB} are attenuated.

Phase Modulation of a PLL

A PLL can be phase modulated by injecting a baseband modulating voltage at the output of the phase detector as shown below.

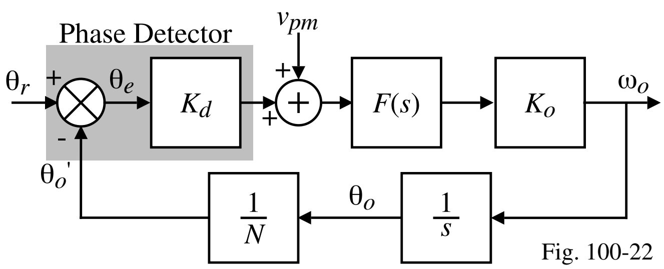


Fig. 100-22

The transfer function from the modulation input, v_{pm}, to the output phase, θ_o, is given as

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{F(s)K_o}{s + \frac{K_v F(s)}{N}}$$

The fundamental behavior of the loop is determined by letting $F(s) = 1$ to get

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{K_o}{s + \frac{K_v}{N}} \quad \rightarrow \quad \text{Lowpass filter with a gain of } \frac{N}{K_d} \text{ and } \omega_{-3dB} = \frac{K_v}{N}$$

Note that modulation frequencies greater than ω_{-3dB} are attenuated.

FM Demodulation using a PLL

The PLL can also serve as an FM demodulator. The following block diagram is a PLL FM receiver.

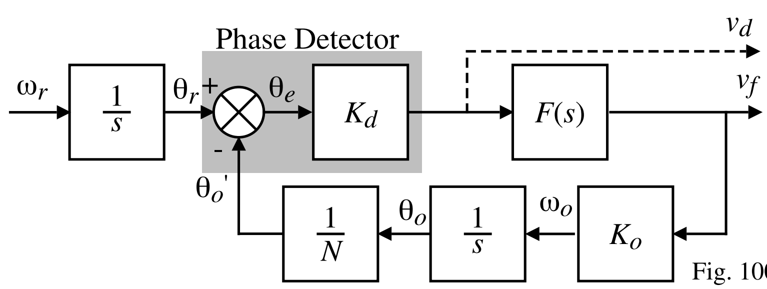


Fig. 100-23

The transfer function from ω_r to the output of the demodulator, v_f, is given as

$$\frac{V_f(s)}{\omega_r(s)} = \frac{K_d F(s)}{s + \frac{K_v F(s)}{N}}$$

The fundamental behavior of the loop is determined by letting $F(s) = 1$ to get

$$\frac{V_f(s)}{\omega_r(s)} = \frac{K_d}{s + \frac{K_v}{N}} \quad \rightarrow \quad \text{Lowpass filter with a gain of } \frac{N}{K_o} \text{ and } \omega_{-3dB} = \frac{K_v}{N}$$

FM Demodulation using a PLL - Continued

Comments:

- Note that the loop demodulates signals *within* the loop bandwidth. This configuration is called a "modulation tracking loop". Within the loop the transfer function is,

$$\frac{V_f(s)}{\omega_r(s)} = \frac{N}{K_o}$$

- The VCO linearity controls the linearity of the FM demodulator. Many applications use $N = 1$.

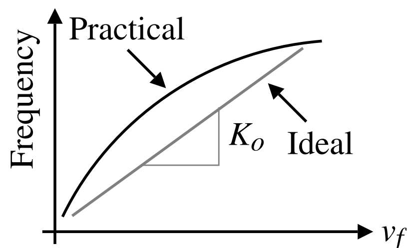


Fig. 100-24

Phase Demodulation using a PLL

The PLL can also be used to demodulate PM.

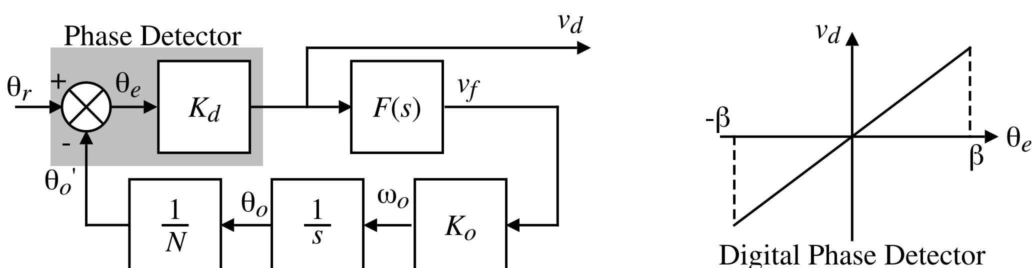


Fig. 100-25

The transfer function from the input phase, θ_r, to the demodulated output, v_d, is

$$\frac{V_d(s)}{\theta_r(s)} = \frac{sK_d}{s + \frac{K_v F(s)}{N}}$$

The fundamental behavior of the loop is determined by letting $F(s) = 1$ to get

$$\frac{V_d(s)}{\theta_r(s)} = \frac{sK_d}{s + \frac{K_v}{N}}$$ → Highpass filter with a gain of K_d and $\omega_{-3dB} = \frac{K_v}{N}$

The PLL acts like a highpass filter and demodulates only those signals above the loop bandwidth. Beyond the loop bandwidth the transfer function is K_d.

Therefore, the phase detector determines the linearity of the PM demodulator.

Phase Demodulation with No Carrier

In many cases, the phase modulation is transmitted without a carrier. An example of BPSK is shown.

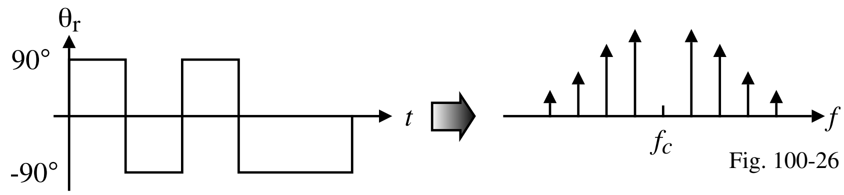


Fig. 100-26

Ordinary PLLs phase lock to the carrier and hence cannot be used to demodulate this type of modulation.

The most common methods for recovering the carrier and demodulating the signal are the *squaring loop*, *remodulator*, and the *Costas loop*.

Phase Demodulation with No Carrier – Squaring Loop

Block diagram of the demodulator:

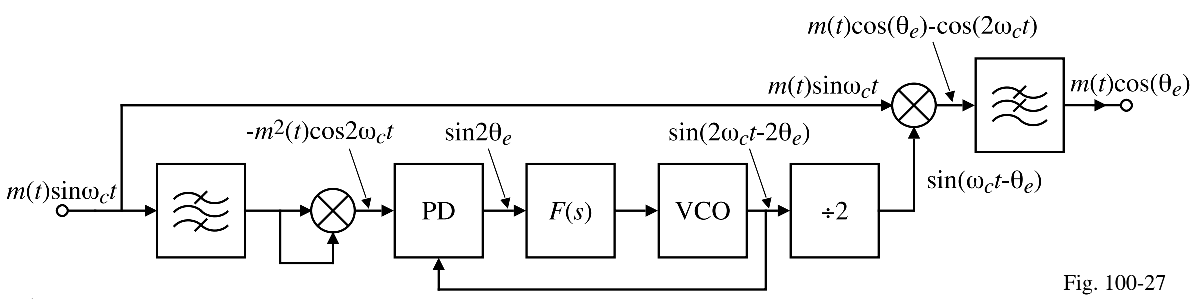


Fig. 100-27

Operation:

1.) The input to the demodulator can be represented as,

$$v_r(t) = m(t)\sin(\omega_c t)$$

which has a zero average value.

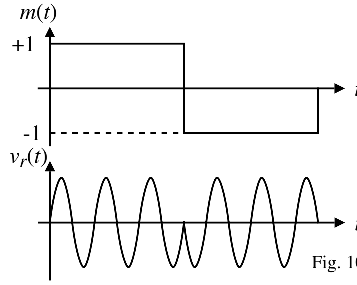


Fig. 100-28

2.) After the squaring function, the signal is

$$0.5m^2(t)[1 - \cos(2\omega_c t)]$$

3.) Dividing by 2 recovers the carrier signal.

4.) The second multiplier and lowpass filter detect the modulation.

The second divider has an ambiguity in its starting state which must be removed by coding or some other means.

Phase Demodulation with No Carrier – Remodulator Loop

One of the problems with the squaring loop is that the carrier frequency is doubled.

The remodulator multiplies the input by $m(t)\cos(\theta_e)$ rather than $m(t)\sin(\omega_c t)$ avoiding the double-frequency carrier.

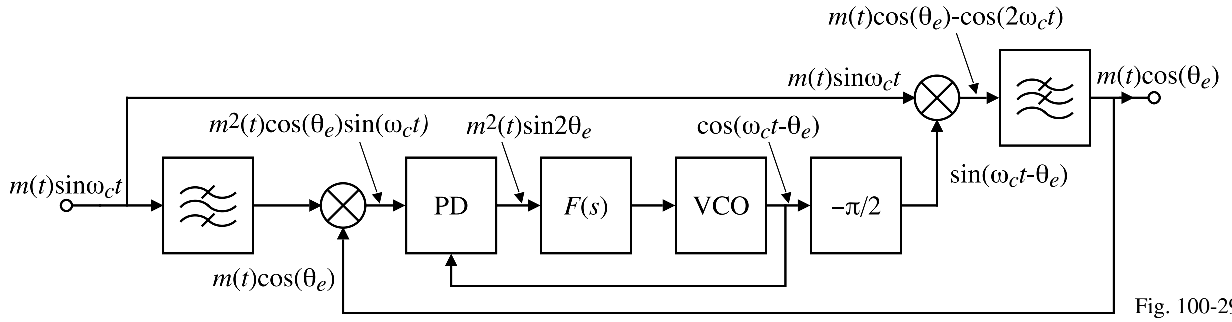


Fig. 100-29

Note that the signal $m(t)$ is modulated onto the carrier at the first multiplier – hence the name remodulator.

Phase Demodulation with No Carrier – Costas Loop

The Costas loop reverses the order of multiplication putting the phase detector before the multiplier.

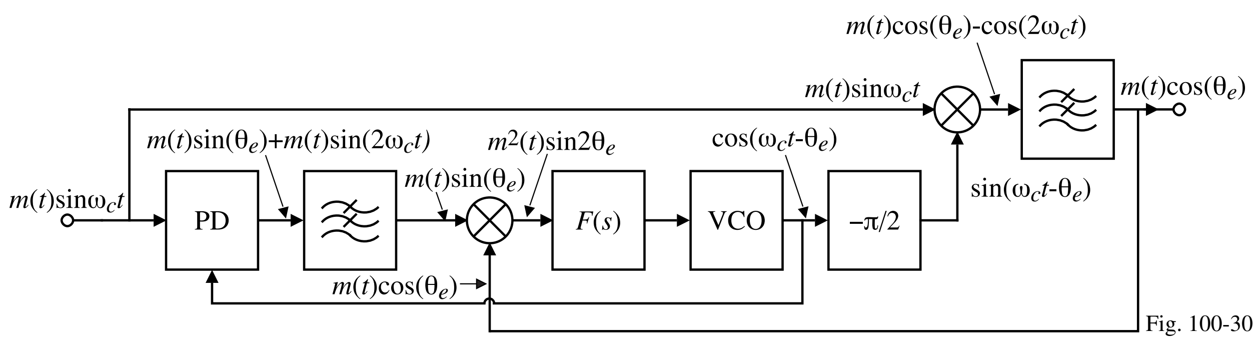


Fig. 100-30

Putting the phase detector before the multiplier replaces the noise-limiting bandpass filter with a simpler lowpass filter.

The phase demodulation schemes presented above (squaring loop, remodulator, and Costas) are only good for binary modulation. Similar but more complex circuits are required for quaternary PSK.

FREQUENCY SYNTHSIZERS

Multi-Loop DPLL Frequency Synthesizer

A multi-loop frequency synthesizer is a way of implementing very small channel requirements without having a small reference frequency.

The two-loop or multi-loop frequency synthesizer can provide the required frequency resolution while meeting the capture time and VCO phase noise specifications.

Two-loop DPLL frequency synthesizer shown has a $f_{-3dB} = 0.1f_{ref}$ and a f_{ref} = 1kHz:

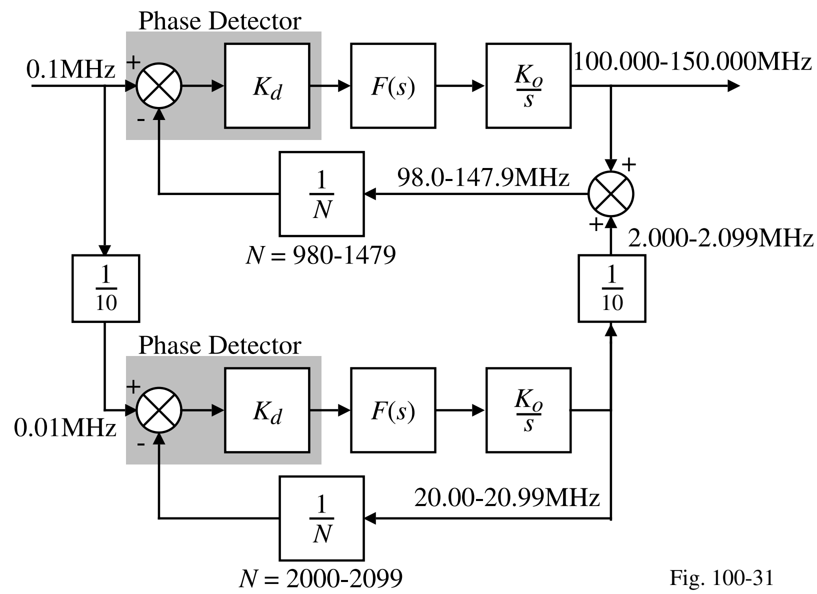


Fig. 100-31

Fractional-N PLL Frequency Synthesizer

The fractional-N PLL has the ability to resolve the channel spacing to less than f_{ref}/N.

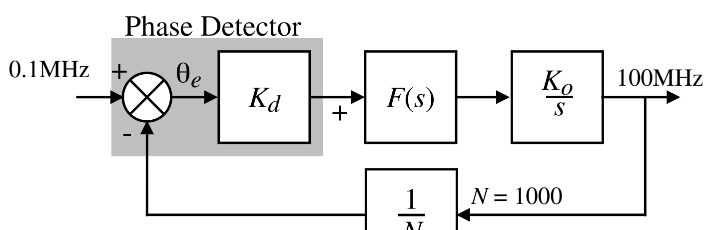


Fig. 100-32

How do you increase the frequency resolution by a factor of 10?

1.) Replace the 0.1MHz reference with a 0.01MHz reference. However, this will slow the loop by a factor of 10.

2.) Use a multi-loop synthesizer.

3.) Use the fractional-N method.

Principle of the fractional-N technique:

Divide by N_1 for P periods and by N_2 for Q periods.

$$\text{Effective } N = \left(\frac{P}{P+Q}\right) N_1 + \left(\frac{Q}{P+Q}\right) N_2 = \frac{PN_1 + QN_2}{P+Q}$$

Fractional-N PLL Frequency Synthesizer - Continued

Comments on the Fractional-N Technique:

- Usually we take $N_2 = N_1+1$ to determine the average output frequency. Therefore,

$$N_{aver} = \frac{PN_1+Q\,(N_1+1)}{P+Q} = N_1+\frac{Q}{P+Q} = N_1+f$$

where f is the frequency step and is $0 \leq f \leq 1$

- Consider the example from the previous slide.

Assume we want a resolution of $f_{ref}/10$. Therefore, $N_1 = 999$, $P = 1$, $N_2 = 1000$, and $Q = 9$. This gives,

$$f_{out} = 0.1\text{MHz}\left(\frac{PN_1 + Q(N_1+1)}{P+Q}\right) = 0.1\left(\frac{1\cdot 999 + 9\cdot 1000}{10}\right)\text{MHz}$$

$$= 0.1\left(\frac{9999}{10}\right)\text{MHz} = 99.99\text{MHz}$$

- Another example follows. Let $P = 5$ and $Q = 5$. The output frequency is

$$f_{out} = 0.1\text{MHz}\left(\frac{PN_1 + Q(N_1+1)}{P+Q}\right) = 0.1\left(\frac{5\cdot 999 + 5\cdot 1000}{10}\right)\text{MHz} = 99.95\text{MHz}$$

By adjusting P and Q we can fill in all 10kHz increments between the 100kHz reference steps.

Fractional-N PLL Frequency Synthesizer - Continued

Implementation of the fractional-N synthesizer:

- The change of one unit in the frequency divider is most often accomplished with the *swallow counter* or *cycle swallower* shown below.

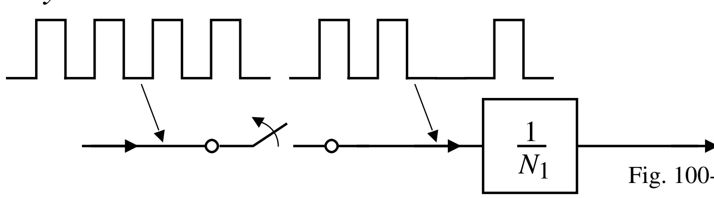


Fig. 100-33

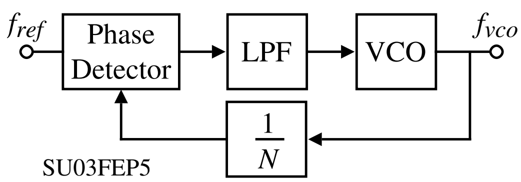
- Upon command, the cycle swallower removes a pulse and thus increases the overall frequency division by one.
- The fractional-N synthesizer method sets the average frequency to the required value. However, the reference frequency is not equal to the feedback frequency.
- Since the reference frequency is not equal to the fedback frequency, there will be a phase jitter that is introduced in the form of spurs

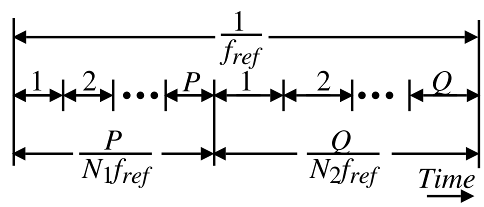
Example 4 – Fractional-*N* Division Ratio

The block diagram for a fractional-N frequency synthesizer is shown. The effective N is normally given as shown below (see page 100-30 of the lecture notes) where the divider divides by N_1 for P cycles (periods) and N_2 for Q cycles (periods).

$$N_{eff} = \frac{PN_1+QN_2}{P+Q}$$

The above relationship is only good when N_{eff} is large. (a.) Derive a better expression for N_{eff} based on the diagram shown which takes into account that f_{ref} always remains constant but that the output frequency of the VCO, f_{VCO}, changes as N changes. (b.) If $P = 1$, $Q = M$-1, $N_1 = N$+1 and $N_2 = N$, find N_{eff} for both the above expression and the one derived in (a.). (c.) If $M = 100$ and $N = 1000$, what is the effective N for both expressions?





Solution

(a.) We know that f_{ref} never changes so $f_{vco1}=N_1f_{ref}$ and $f_{vco2}=N_2f_{ref}$ also $T_{vco1} = 1/f_{vco1}$ and $T_{vco2} = 1/f_{vco2}$. We can express the average clock period, T_{vco}(aver) as

Example 4 - Continued

$$T_{vco}(\text{aver}) = \frac{T_{vco1}P+T_{vco2}Q}{P+Q} = \frac{\frac{P}{f_{vco1}}+\frac{Q}{f_{vco2}}}{P+Q} = \frac{\frac{P}{N_1}+\frac{Q}{N_2}}{(P+Q)f_{ref}}$$

$$\therefore\ f_{vco}(\text{aver}) = \frac{1}{T_{vco}(\text{aver})} = \frac{(P+Q)f_{ref}}{\frac{P}{N_1}+\frac{Q}{N_2}} = N_{eff}f_{ref} \rightarrow \boxed{N_{eff} = \frac{(P+Q)}{\frac{P}{N_1}+\frac{Q}{N_2}} = \frac{N_1N_2(P+Q)}{N_2P+N_1Q}}$$

(b.) $$N_{eff1} = \frac{PN_1+QN_2}{P+Q} = \frac{(N+1)+N(M-1)}{M} = N+\frac{1}{M}$$

$$N_{eff2} = \frac{N_1N_2(P+Q)}{N_2P+N_1Q} = \frac{N(N+1)M}{N+(N+1)(M-1)} = \frac{NM(N+1)}{M(N+1)-1} = \frac{N}{1-\frac{1}{M(N+1)}} \approx N+\frac{N}{M(N+1)}$$

$$\therefore\ \boxed{N_{eff1} = N+\frac{1}{M} \quad \text{and } N_{eff2} = \frac{N}{1-\frac{1}{M(N+1)}} \approx N+\frac{N}{M(N+1)}}$$

(c.) $\boxed{N_{eff1} = 1000.01 \quad \text{and } N_{eff2} = 1000.00999}$

Fractional-N PLL Frequency Synthesizer - Continued

The problem of spurs in a fractional-*N* PLL:

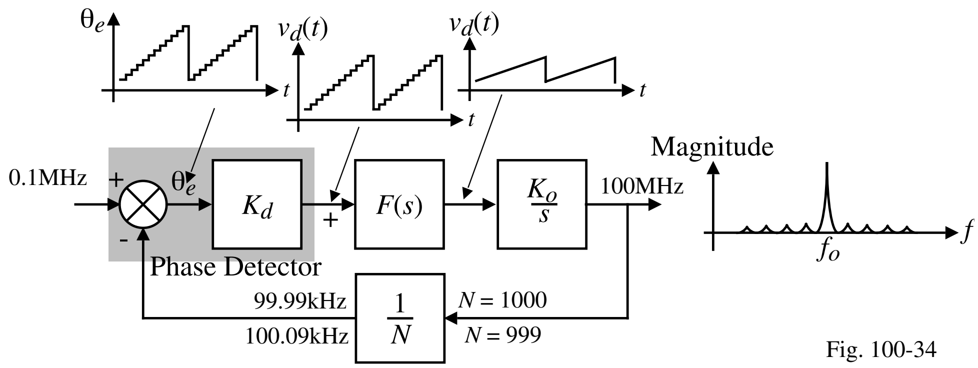


Fig. 100-34

How do the spurs occur?

1.) Assume that the VCO output frequency is the desired 99.99MHz.

2.) 99.99MHz divided by 1000 puts 99.99kHz to the phase detector.

3.) The phase detector generates a phase error in the direction to increase the VCO frequency.

4.) The phase error accumulates until the divider changes to 999.

5.) Then the phase error is reversed and causes the VCO frequency to decrease.

6.) The result of this behavior is a sawtooth ripple on the phase detector output voltage.

7.) If not removed, this ripple would cause severe spurs in the output of the synthesizer.

Fractional-N PLL Frequency Synthesizer - Continued

A method to remove the spurs from the Fractional-*N* synthesizer:

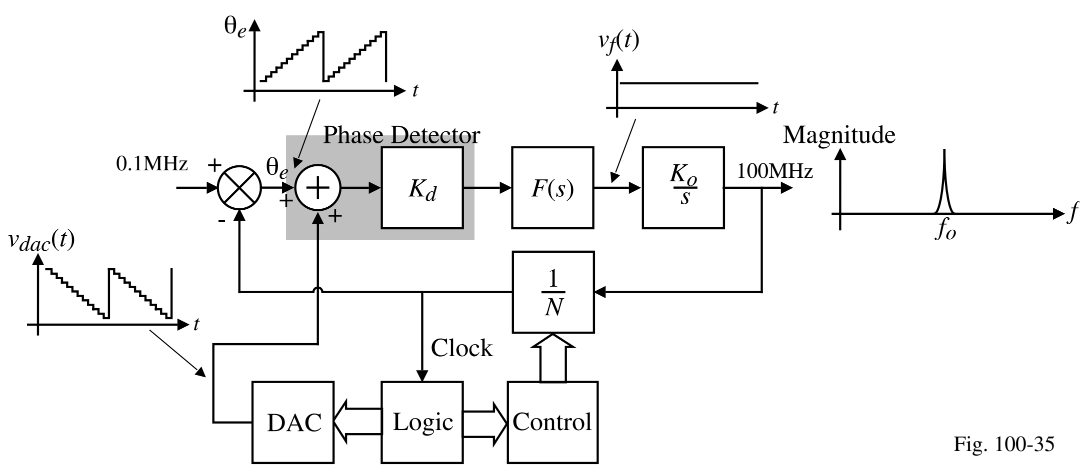


Fig. 100-35

The above circuit uses a DAC to create an inverse phase-voltage to keep the control voltage to the VCO flat.

Comments:

- By increasing *P* and *Q* it is possible to greatly increase the frequency resolution.
- The phase noise performance depends upon how well the DAC removes the reference noise sidebands.

CONTINUATION OF THE 450MHZ FREQUENCY SYNTHESIZER EXAMPLE

Specifications

Design a DPLL frequency synthesizer that meets the following specifications:

Frequency Range:	450 – 475 MHz
Channel Spacing:	25 kHz
Modulation:	FM from 300 to 3000 Hz
Modulation Deviation:	±5kHz
Loop Type:	Type 2
Loop Order:	Second order
VCO Gain:	K_o = 1.25MHz/V = 7.854 Mradians/sec./V
Phase Detector Type:	PFD ($\beta = 2\pi$)
Phase Detector Gain:	K_d = 0.796 V/radian
Reference Frequency	
FM Spurs:	*< -70 dBc*
Prescaler:	*20/21 Dual Modulus*

Reference Frequency Ripple Voltage Modulation of the VCO

Since the ripple on the VCO control voltage is the same as PM, we can use the previous results developed for PM.

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{F(s)K_o}{s + \frac{K_v F(s)}{N}}$$

The filter was given as,

$$F(s) = \frac{sR_2C + 1}{sR_1C} = \frac{s\tau_2 + 1}{s\tau_1} \quad \text{where } \tau_1 = 0.419 \text{ ms} \quad \text{and} \quad \tau_2 = 1.575 \text{ ms}$$

The closed-loop transfer function is given as,

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{\left(\frac{s\tau_2 + 1}{s\tau_1}\right)K_o}{s + \frac{K_v\left(\frac{s\tau_2 + 1}{s\tau_1}\right)}{N}} = \frac{K_o\left(\frac{\tau_2}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right)}{s^2 + \left(\frac{K_v}{N}\right)\left(\frac{\tau_2}{\tau_1}\right)s + \frac{K_v}{N\tau_1}} = \frac{K_o\left(\frac{\tau_2}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

At high frequencies (greater than the closed-loop bandwidth), the transfer function becomes,

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{K_o}{s}\left(\frac{\tau_2}{\tau_1}\right) \quad \rightarrow \quad \theta_o = \frac{K_o}{s}\left(\frac{\tau_2}{\tau_1}\right)V_{pm}$$

Therefore, the reference frequency ripple voltage present at the input of the loop filter causes PM spurs to appear at the output.

Source of Reference Frequency Modulation

One of the more significant sources of reference frequency modulation comes from the input offset voltage of the filter if op amps are used.

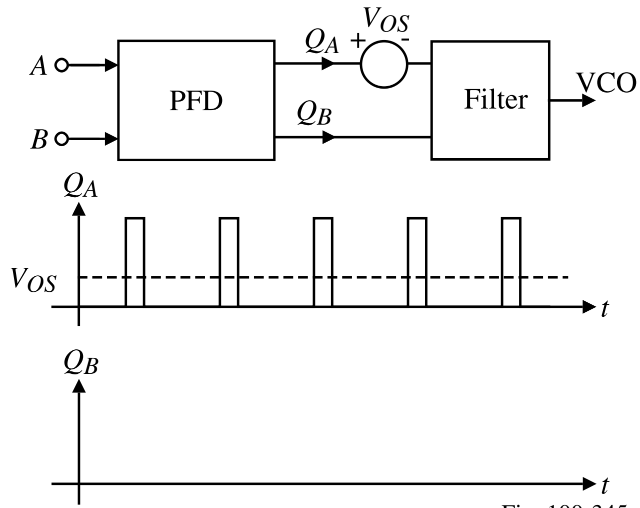


Fig. 100-345

- The level of offset voltage should be much less than 1mV for most applications.
- The dc offset will cause a single sideband, sinusoidal phase modulation of twice the dc offset voltage, V_{OS}.

Source of Reference Frequency Modulation - Continued

Consider the dc offset of the active filter implementation.

The offset voltage at the input of the filter is

$$V_{OS} = I_{os}R_1 + V_{io} + I_{os}R_1 = V_{io} + 2I_{os}R_1$$

Using worst case data for the OP-27,

$$I_{os} = 50\text{nA} \quad \text{and} \quad V_{io} = 60\mu\text{V}$$

$$\therefore \quad V_{OS} = 60\mu\text{V} + 2\cdot 50\text{nA}\cdot 2.4\text{k}\Omega = 300\mu\text{V}$$

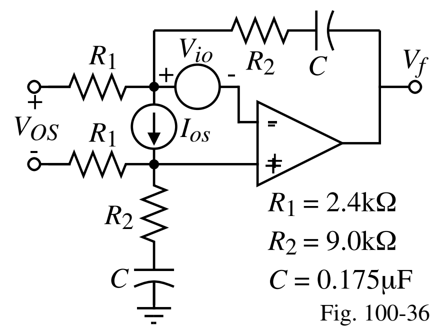


Fig. 100-36

Assume that the OP-27 can be trimmed so that the offset is 18µV. The first ac harmonic is twice the DC value (one sideband only) giving,

$$V_{pm} = 2V_{OS} = 36\mu\text{V}$$

The spurious deviation due to the offset voltage is

$$\theta_d = \frac{K_o}{s}\left(\frac{\tau_2}{\tau_1}\right) V_{pm} = \frac{K_o}{\omega_m}\left(\frac{\tau_2}{\tau_1}\right) V_{pm} = \frac{7.854\text{x}10^6}{2\pi\cdot 25\text{x}10^3}\left(\frac{1.545}{0.419}\right) 36\mu\text{V} = 6.64\text{x}10^{-3} \text{ radians}$$

$\therefore$ The spur at the reference frequency (25kHz) is

$$SSB = 20\log_{10}\left(\frac{\theta_d}{2}\right) = 20\log_{10}\left(\frac{6.64\text{x}10^{-3}}{2}\right) = -49.6 \text{ dBc}$$

Source of Reference Frequency Modulation – Continued

Applying $v_{pm}/2$ = 18µV to the reference modulation transfer function gives the following result. The spurs created by the reference and its harmonics can be read from the graph.

PSPICE Input File:

```
450-475MHz DPLL Design Problem-Spurs for 2nd order filter
.PARAM N1=18000, N2=19000, KVCO=7.854E6, T1=0.419E-3
.PARAM T2=1.575E-3, KD=0.796, E=0.001
VS 1 0 AC 18.0E-6
R1 1 0 10K
ELPLL 2 0 LAPLACE {V(1)}=
+{KVCO*T2/T1*(S+1/T2)/(S*S+KD*KVCO*T2/T1/N1*S+KD*KVCO/N1/T1)}
R2 2 0 10K
*Steady state AC analysis
.AC DEC 20 1 1000K
.PRINT AC VDB(2) VP(2)
.PROBE
.END
```

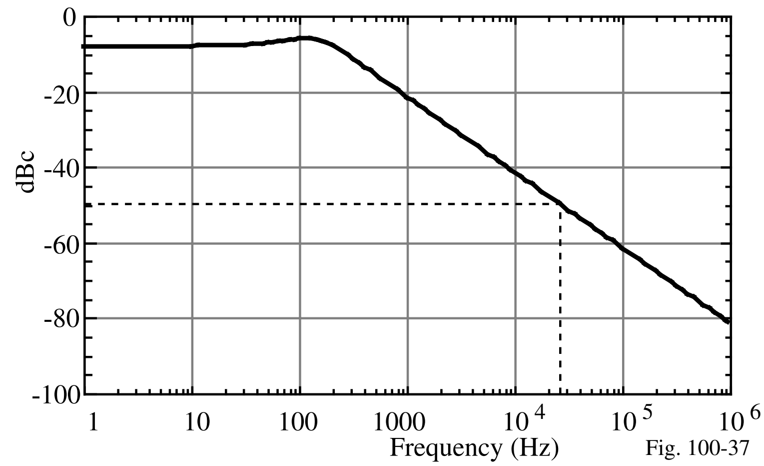


Fig. 100-37

Redesign with a Third-Order Loop

Because the spur specification is not met, we will design a third-order filter to get the additional attenuation needed.

Third-Order Filter:

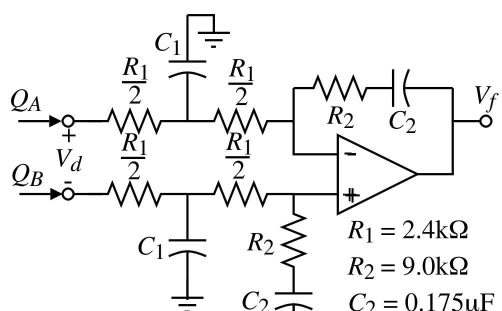


Fig. 100-38

The passive pre-filter will provide the additional attenuation. It also has the effect of increasing the rise and fall times of the pulsed input to the op amp which reduces the slew-rate requirements of the op amp.

To achieve the –70dBc spur level we need 20.6dB of additional attenuation. Since we want 20.6dB of additional attenuation at 25kHz, find the pole frequency of the filter.

$$\text{No. of decades} = 10^{\frac{\text{Attenuation}}{20}} = 10^{\frac{20.6}{20}} = 1.03 \text{ decades}$$

$$f_c = \frac{f_r}{10^{\text{ndec}}} = \frac{f_r}{10^{1.03}} = \frac{25\text{kHz}}{10.715} = 2.333\text{kHz}$$

It can be shown that $C_1 = \dfrac{4}{R_1 2\pi f_c} = 0.114\mu\text{F}$

Third-Order Reference Modulation Transfer Function

The new filter function can be expressed as,

$$F(s) = \frac{s\tau_2 + 1}{s\tau_1}\left(\frac{1}{s\tau_3+1}\right)$$

The transfer function from the reference modulation to the output phase is

$$\frac{\theta_o(s)}{V_{pm}(s)} = \frac{\left(\frac{s\tau_2 + 1}{s\tau_1(s\tau_3+1)}\right)K_o}{s + \frac{K_v\left(\frac{s\tau_2+1}{s\tau_1(s\tau_3+1)}\right)}{N}} = \frac{K_o\left(\frac{\tau_2}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right)}{s^3\tau_3 + s^2 + \left(\frac{K_v}{N}\right)\left(\frac{\tau_2}{\tau_1}\right)s + \frac{K_v}{N\tau_1}}$$

Spur Performance for the Third-Order Filter

Applying $v_{pm}/2$ = 18mV to the reference modulation transfer function as before give the following results.

PSPICE File:

```
450-475MHz DPLL Design Problem-Spurs for 3rd order filter
.PARAM N1=18000, N2=19000, KVCO=7.854E6, T1=0.419E-3
.PARAM T2=1.575E-3, KD=0.796, E=0.001, T3=6.822E-5
VS 1 0 AC 18.0E-6
R1 1 0 10K
ELPLL2O 2 0 LAPLACE {V(1)}=
+{KVCO*T2/T1*(S+1/T2)/(S*S+KD*KVCO*T2/T1/N1*S+KD*KVCO/N1/T1)}
R2 2 0 10K
ELPLL3O 3 0 LAPLACE {V(1)}=
+{KVCO*T2/T1*(S+1/T2)/(S*S*S*T3+S*S+KD*KVCO*T2/T1/N1*S+KD*KVCO/N1/T1)}
R3 3 0 10K
*Steady state AC analysis
.AC DEC 20 1 1000K
.PRINT AC VDB(2) VDB(3)
.PROBE
.END
```

Results:

The spur specification is exactly satisfied.

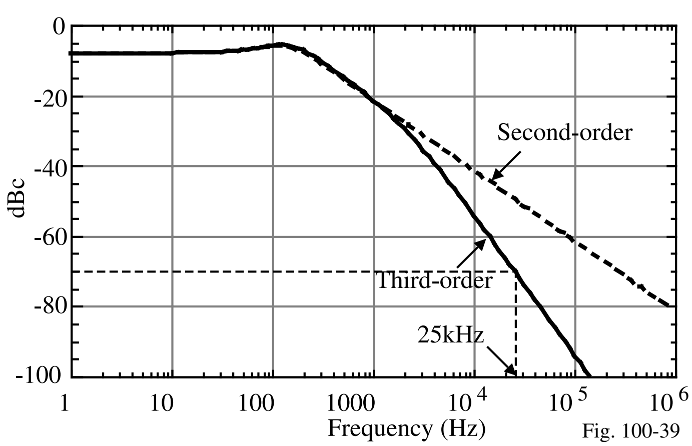


Fig. 100-39

Stability of the Third-Order Loop

The loop gain of the third-order PLL is given by,

$$LG(s) = \frac{s\tau_2 + 1}{s^2\tau_1}\left(\frac{K_dK_o}{N(s\tau_3+1)}\right)$$

Using the previously designed parameters we get the open-loop gain below.

PSPICE File:

```
450-475MHz DPLL Design Problem-Spurs for 3rd order filter
.PARAM N1=18000, N2=19000, KVCO=7.854E6, T1=0.419E-3
.PARAM T2=1.575E-3, KD=0.796, E=0.001, T3=6.822E-5
VS 1 0 AC 1
R1 1 0 10K
ELPLL3ORDER 2 0 LAPLACE {V(1)}
+={KD*KVCO*(S*T2+1)/((S+E)*N1*T1*(S+E)*(1+S*T3))}
R2 2 0 10K
*Steady state AC analysis
.AC DEC 20 1 1000K
.PRINT AC VDB(2) VP(2)
.PROBE
.END
```

Plot:

Phase margin ≈ 62°

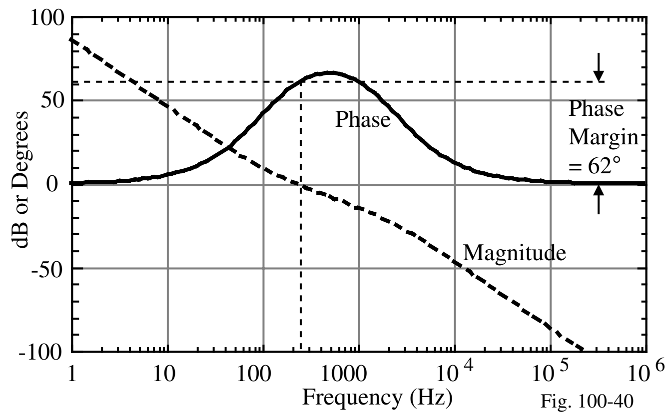


Fig. 100-40

VCO Phase Noise

The third-order transfer function from the VCO phase noise to the output is,

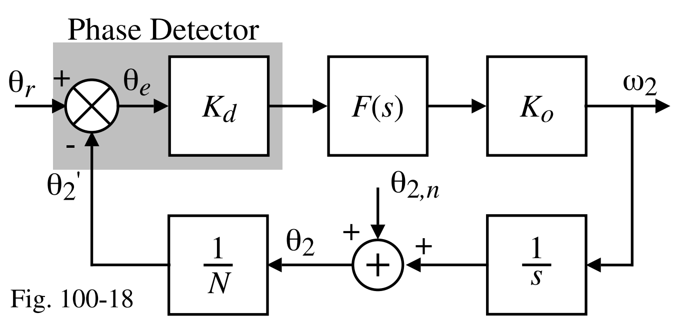


Fig. 100-18

The transfer function from the VCO noise, $\theta_{o,n}$ to the output, θ_2, is given as

$$\frac{\theta_2'(s)}{\theta_{o,n}(s)} = \frac{\frac{s}{N}}{s + \frac{K_vF(s)}{N}} = \frac{\frac{s}{N}}{s + \frac{K_v\left(\frac{s\tau_2+1}{s\tau_1}\right)\left(\frac{1}{s\tau_3+1}\right)}{N}} = \frac{s^3\tau_3 + s^2}{s^3\tau_3 + s^2 + s\frac{K_v}{N}\frac{\tau_2}{\tau_1} + \frac{K_v}{N\tau_1}}$$

VCO Phase Noise - Continued

Assume that the phase noise is given as,

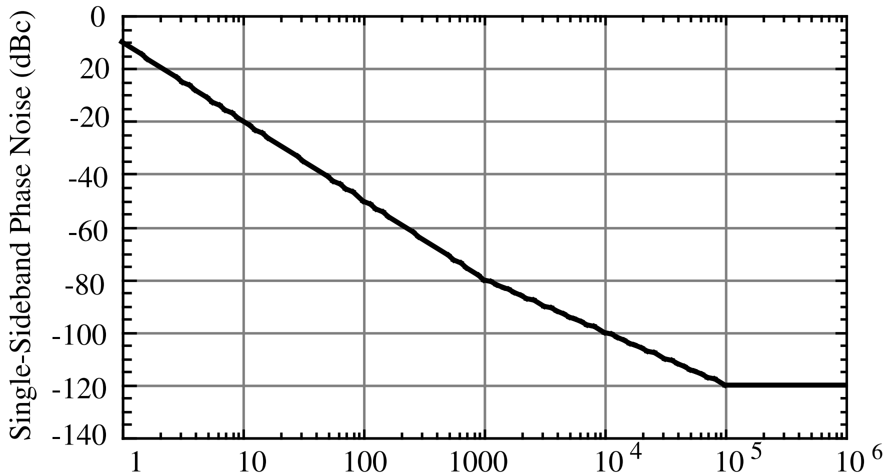


Fig. 100-41

How do you model this noise source in PSPICE?

```
vphasenoise 1 0 ac 1.0
R1 1 0 10k
EPN 2 0 freq {v(1)} = (1,10,0) (10,-20,0) (100, -50,0) (1000,-80,0) (10000,-100,0)
+(100000,-120,0) (1E6,-120,0)
RL 2 0 10k
```


VCO Phase Noise - Continued

The plot below shows the VCO phase noise, the closed-loop transfer function, and the VCO phase noise at the output of the loop.

PSPICE File:

```
450-475MHz DPLL Design Problem-In/Out VCO Phase Noise, Transfer Function
.PARAM N1=18000, N2=19000, KVCO=7.854E6, T1=0.419E-3
.PARAM T2=1.575E-3, KD=0.796, E=0.001, T3=6.822E-5
*Input Phase Noise
vphasenoise 1 0 ac 1.0
R1 1 0 10k
EPN 2 0 freq {v(1)} = (1,10,0) (10,-20,0) (100, -50,0) (1000,-80,0)
+(10000,-100,0) (100000,-120,0) (1E6,-120,0)
RPN 2 0 10k
*DPLL Transfer Function
EDPLL1 3 0 LAPLACE {V(1)}=
+{S*S*(T3*S+1)/(S*S*S*T3+S*S
++KD*KVCO*T2/N1/T1*S+KD*KVCO/N1/T1)}
RDPLL1 3 0 10K
*VCO Noise at the Output
EDPLL2 4 0 LAPLACE {V(2)}=
+{S*S*(T3*S+1)/(S*S*S*T3+S*S
++KD*KVCO*T2/N1/T1*S+KD*KVCO/N1/T1)}
RDPLL2 4 0 10K
*Steady state AC analysis
.AC DEC 20 1 1000K
.PRINT AC VDB(2) VDB(3) VDB(4)
.PROBE
.END
```

Plot:

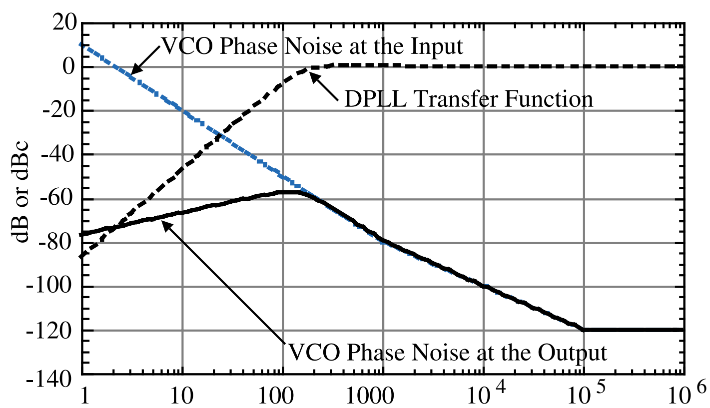


Fig. 100-42

Lock Range, Lock Time and Pull-In Range for the 450MHz Frequency Synthesizer Example

Lock Range:

$$\Delta\omega_L = 2N\beta\zeta\omega_n = (2)(18{,}000)(2\pi)(0.726)(905) = 148.62 \text{ Mradians/sec.}$$

$$\Delta f_L = 23.65 \text{ MHz}$$

Lock Time:

$$T_L \approx \frac{1}{\omega_n} = \frac{1}{905} = 1.1 \text{ msec.}$$

Pull-In Range:

The pull-in range is theoretically infinite. It will be limited by the dynamic range of the loop components.

Prescaler Design

The equations needed to fine the M- and A-counter values are:

$$f_o = [(M\text{-}A)P + (P+1)A]f_r$$

$$N = \frac{f_o}{f_r} = (M\text{-}A)P + (P+1)A = MP+A$$

$$\frac{N}{P} = M + \frac{A}{P} \qquad \text{where} \quad M = \text{integer of}\left(\frac{N}{P}\right) \qquad \text{and } A = N\text{-}MP$$

For this example, we find the values of M and A to produce an output frequency of 451.075MHz.

$$N = \frac{f_o}{f_r} = \frac{451.075 \text{ MHz}}{0.025 \text{ MHz}} = 18043$$

$$\therefore \quad M = \text{integer of}\left(\frac{N}{P}\right) = \frac{18043}{20} = 902.15 = 902 \quad \text{and } A = N\text{-}MP = 18043\text{-}902\cdot 20 = 3$$

Other values are calculated in a similar manner:

M	MP	A				
	$P = 20$	0	1	2	···	19
900	18000	450.0	450.025	450.050	$(MP+A)\,f_r$	450.475
901	18020	450.5	450.525	450.550	···	450.975
···	···	···	···	···	···	···
949	18980	474.5	474.525	474.550	···	474.975

SUMMARY

- Review of Modulation
  - AM, FM, and PM
  - Spurs
  - Influence of frequency multiplication and division on spurs
- Phase Noise
  - Single sideband noise
  - Reference phase noise
  - VCO phase noise
- Use of a PLL for Modulation and Demodulation
- Frequency Synthesizers
  - Achieving small channel resolution without using small reference frequencies
    - - Multi-loop
    - - Fractional *N*
- This ends the system level perspective of PLLs

